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PERIODIC GROUND STATES AND THE MANY-BODY PROBLEM*

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In a recent Letter, Overhauser[1] proposed that the description of the ground state of nuclear matter start with spatially periodic Hartree-Fock functions. The periodic state has been proposed in other many-body problems, and may occur widely in nature. Examples that have been worked out in some detail are Vlasov's classical theory of "solids,"[2] Fröhlich's one-dimensional model of superconductivity,[3] and the author's theory of interacting bosons.[4] The physical idea of these theories is that when attractive interactions outweigh repulsions at a characteristic distance, the system may pay the cost in kinetic energy to establish space periodicity. Except for close collisions, a particle moves freely through the entire volume of the system, in contrast to the localization in a solid state. Yet there is long-range order involving a self-consistent density variation. This Letter first reports a general method to describe systems in which the periodic state forms the correct starting point. It enables one to determine the excitations and thermodynamic properties, as well as to calculate modifications of the self-consistent field produced by close collisions. Second, we note some qualitative and unusual features of such periodic states. These have been studied in some detail in I and II for bosons. Since the present approach embraces both fermions and bosons, the features are general possibilities. Because of the difficulties of making quantitative predictions in many-body theory, the qualitative ideas suggest a general search for experimental evidence for the features.

Consider, for simplicity, spinless bosons or fermions, described by a quantized field $\psi(\vec{x}) = \sum_f a_f \varphi_f(\vec{x})$. The $\varphi_f(\vec{x})$ form a complete set of one-body functions; we are mainly concerned with a plane wave basis or a set of solutions to a periodic potential problem in extended zone correspondence with the plane waves. The basic units are the operators $c(\vec{f}\,|\,\vec{f}\,') = a_f^\dagger a_{f'}$, i.e., components of the density matrix $\rho(\vec{x}, \vec{y}) = \psi^\dagger(\vec{x})\psi(\vec{y})$. They obey the commutation rules

$$[c(\vec{f}\,|\,\vec{f}\,'), c(\vec{g}\,|\,\vec{g}\,')] = c(\vec{f}\,|\,\vec{g}\,')\delta_{f',g} - c(\vec{g}\,|\,\vec{f}\,')\delta_{f,g'},$$

for both Bose and Fermi systems. The Hamiltonian is

$$H = \sum_{f,g} T(\vec{f}\,|\,\vec{g})c(\vec{f}\,|\,\vec{g}) + \sum_{f,g,f',g'} V(\vec{f}\vec{g}\,|\,\vec{f}\,'\vec{g}\,')c_2(\vec{f}\vec{g}\,|\,\vec{f}\,'\vec{g}\,'),$$

with

$$T(\vec{f}\,|\,\vec{g}) = (\hbar^2/2M)\int \nabla\varphi_f^* \cdot \nabla\varphi_g \, d^3x,$$

$$V(\vec{f}\vec{g}\,|\,\vec{f}\,'\vec{g}\,') = \tfrac{1}{2}\iint \varphi_f^*(\vec{x})\varphi_g^*(\vec{y})V(\vec{x}-\vec{y})\varphi_{f'}(\vec{x})\varphi_{g'}(\vec{y})\, d^3x\, d^3y,$$

$$c_2(\vec{f}\vec{g}\,|\,\vec{f}\,'\vec{g}\,') = a_f^\dagger a_g^\dagger a_{f'} a_{g'}.$$

c_2 can be broken into products of c operators. The $c(\vec{f}\,|\,\vec{f}\,')$ thus form a complete description.

The elementary Hartree theory consists in

assigning values to the number operators $c(\vec{f}|\vec{f})$, neglecting off-diagonal elements and contributions of $c(\vec{f}|\vec{f}')$ with $\vec{f}' \neq \vec{f}$. The choice $c(0|0)=N$, $c(\vec{f}|\vec{f})=0$ for $\vec{f}\neq 0$ yields the semi-classical boson theory of I. $c(\vec{f}|\vec{f})=1$ for $|\vec{f}|$ less than the wave vector in correspondence with the Fermi momentum, for fermions leads, for example, to Overhauser's ground state for nuclear matter. One obtains a set of equations for $\varphi_f(\vec{x})$ by functionally varying the energy with respect to $\varphi_f^*(\vec{x})$. The plane wave basis makes the energy stationary, but when attractive forces are vital, a spatially periodic basis yields a minimum energy.

It is important that there are other periodic solutions of the Hartree equations (different lengths or symmetries) of higher energy than the lowest solution, but lower in energy than the plane wave solutions. As discussed in II, the various periodic states are mutually orthogonal as $N\to\infty$. They have a different energy per particle and so determine the properties at a finite temperature. In fact, the correlation function should be periodic at absolute zero and lose the long-range order, as the temperature is raised, because of admixture of other periodicities.

To describe the low-lying excitations, one linearizes the equations of motion for $c(\vec{f}|\vec{f}')$ by breaking down $c_2(\vec{f}\vec{g}|\vec{f}'\vec{g}')$ and retaining only terms containing one number operator. This is a "random phase" approximation, but it is important that it is basis dependent. We obtain a set of linear equations for $c(\vec{f}|\vec{f}')$, an associated excitation spectrum, and a lowering of the ground-state energy from the zero-point shift of the excitations. When one uses a plane wave basis, the resulting theory includes Bogolyubov's theory of bosons and Sawada's theory of fermions. For periodic bases there are new results, most importantly, gaps in the excitation spectra at absolute zero. With rising temperature the other periodic solutions, with gaps in their excitation spectra at different places, come into play. The observed gaps should therefore narrow and ultimately disappear. This effect should be looked for, e.g., by studying inelastic neutron scattering at very low temperatures in liquid helium.

To include the effects of close collisions, c_2 is left unaltered in the equation of motion of c. One writes an equation for the time dependence of c_2. This involves $c_3(\vec{f}\vec{g}\vec{h}|\vec{f}'\vec{g}'\vec{h}')$, which is broken to a sum of products of c_2 and c. Terms containing vacuum expectation values of either of the operators are retained. Elimination of c_2 leads to a new, consistent field as modified by collisions. The above scheme is in the spirit of much recent work in many-body theory. The main point of difference is the recognition of the importance of a non-plane-wave basis even in absence of external fields.

One objects immediately that this violates the translational invariance of the Hamiltonian. For every basis $\varphi_f(\vec{x})$, we get a corresponding theory for a basis $\varphi_f(\vec{x}+\vec{\epsilon})$, where $\vec{\epsilon}$ is any real vector. This point has been studied in II. There, a formalism involving linear combinations of creation and annihilation operators was used. A translationally invariant ground state is constructed by superposing wave functions for different $\vec{\epsilon}$. The ground-state energy per particle is unaltered as $N\to\infty$. Corresponding results are obtained for fermions using Bogolyubov's variational method.[5]

A second point is that the periodic state is not a solid. The solid[4] is described by using a set of $\varphi_f(\vec{x})$, localized at different points in space, and can be treated in the above scheme. A check is obtained by examining the excitation spectrum for long waves. The solid has transverse branches in contrast to the periodic state under consideration. By rewriting the theory in terms of the Wigner distribution, one can show that Vlasov's theory[2] is not a theory of a solid. It is interesting that the solid also has periodic solutions with different periodicities from the one realized at absolute zero. In accordance with the above ideas, these solutions are to be used in the theory of the thermal expansion, for example, of solid He^4.

The above scheme has been used by the author and Vail[6] to extend Fröhlich's one-dimensional model of superconductivity. To Fröhlich's consideration of the static self-consistent state one adds the dynamical interactions of electrons and lattice. This extends the validity of his theory to more reasonable coupling strengths. Fröhlich places the wave vector of the periodicity at twice the Fermi momentum. In three dimensions it is not likely that one can gain energy with such a small periodicity because of the irregular shape of the Fermi surface. But the possibility of a range of long-wavelength periodicities has not been sufficiently explored. This leads back to earlier, abortive theories of superconductivity referred to by Bardeen.[7] Even if the self-consistent superlattice concept is com-

pletely irrelevant to superconductivity, it may be needed in other solid state problems. In particular, Slater[8] has used the idea in an interesting way to give a qualitative explanation of a number of anomalous effects. The self-consistent periodic state is such a general notion that experimental investigation, further exploration of its limitations, or perhaps theoretical refutation seems to be called for.

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LINKED CLUSTER EXPANSION APPLICABLE TO NONSPHERICAL SITUATIONS*

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The Brueckner-Goldstone[1] or linked cluster expansion for the ground-state energy of a many-fermion system has recently been criticized by Kohn and Luttinger[2] and Luttinger and Ward,[3] who have shown by using statistical mechanics, that the formula obtained starting from a spherical unperturbed state is correct, in general, only for systems with complete spherical symmetry. It is the purpose of this note to exhibit a generalized linked cluster expansion applicable also to situations lacking such symmetry.

To illustrate our viewpoint, we consider for simplicity a system of spinless fermions interacting via two-body forces as described by the Hamiltonian

$$H = H_0 + \lambda H_1$$
$$= \sum_{\vec{p}} a^\dagger(\vec{p}) a(\vec{p}) p^2$$
$$+ \frac{\lambda}{2\Omega} \sum_{\vec{p},\vec{p}',\vec{q}} a^\dagger(\vec{p}+\vec{q}) a^\dagger(\vec{p}'-\vec{q}) a(\vec{p}') a(\vec{p}) v(\vec{q}), \quad (1)$$

where p^2 is the kinetic energy of a free particle, Ω is the volume of the system, and the potential $v(\vec{q})$ is assumed not to be spherically symmetric. As is customary, we shall be interested in a system of N particles in the limit in which N and $\Omega \to \infty$, but, of course, $\rho = (N/\Omega)$ remains fixed and finite.

It is first essential to inquire about which element of the Goldstone proof can fail in the present situation. Putting aside questions of convergence, direct examination shows that the formal derivation can fail only if we start the adiabatic switching-on process with an unperturbed state which is orthogonal to the actual ground state. The procedure described below is designed specifically to avoid this dilemma and to yield a series which should represent the lowest normal state.

Let $M(\vec{p}, p_0, \rho)$ be the proper self-energy operator[4] for a single fermion of four-momentum $(\vec{p}, p_0)$ moving through the medium at density ρ. For a normal system, the minimum single-particle excitation energy is required to coincide with the chemical potential, μ, and to be the unique real solution of the equation[5]

$$\mu = p^2 - M(\vec{p}, \mu; \rho). \quad (2)$$

Since in our present example M depends on the direction of $\vec{p}$, for given μ, the real solutions $\vec{p} = \vec{p}_F$ of Eq. (2) define a surface in $\vec{p}$ space, $\mu = \epsilon(\vec{p}_F)$, the perturbed Fermi surface. It is essential for our further considerations to suppose that ρ is itself a single-valued functional of this surface. We may thus write

$$\epsilon(\vec{p}) = p^2 - M(\vec{p}, \epsilon(\vec{p}), \rho[\vec{p}])$$
$$\equiv p^2 + U(\vec{p}), \quad (3)$$

where we have dropped the subscript F to emphasize that we now wish to consider the

family of actual Fermi surfaces which characterize the normal ground states of the Hamiltonian (1) as we vary the density ρ.

From the definition of the chemical potential as the binding energy of the last particle at fixed volume, it now follows that the ground-state energy is given by the expression

$$E = \sum_{\epsilon(\vec{p})<\mu} \epsilon(\vec{p}) = \frac{\Omega}{(2\pi)^3}\int d\omega \int_0^{p_F(\omega)} [p^2 + U(\vec{p})] p^2 dp, \tag{4}$$

where ω indicates the direction in $\vec{p}$ space. Since Eq. (4) has the obvious physical interpretation that we have accumulated the total energy by successively adding particles to the system at fixed Ω, each at the appropriate chemical potential, the total number of particles, N, is, in turn, represented by the expression

$$\rho = (N/\Omega) = \frac{1}{\Omega}\sum_{\epsilon(p)<\mu} 1 = \frac{1}{(2\pi)^3}\int d\omega \int_0^{p_F(\omega)} p^2 dp, \tag{5}$$

which defines the functional $\rho[p_F]$ of whose existence we have already made use.

Equations (4) and (5) further imply the existence of a model Hamilton, H_m,

$$H_m = \sum_{\vec{p}} a^\dagger(\vec{p}) a(\vec{p}) [p^2 + U(\vec{p})]$$

$$\equiv H_0 + V(\lambda), \tag{6}$$

whose ground state coincides with that of the actual system at every density. The significance of H_m for our purposes is that its ground state at the given density is guaranteed not to be orthogonal to the actual ground state. That this is so can be understood by recalling a theorem of Midgal and Galitski,[6] which requires that the momentum distribution of a normal ground state have a discontinuity at the actual Fermi surface. It is clear that a state with this property can be constructed by superposing an unperturbed state which consists of particles filled in up to the actual Fermi surface, i.e., the ground state of H_m, with states coupled to it by the potential, λH_1.

To actually determine E now, we write

$$H = H_m + \lambda H_1 - V(\lambda), \tag{7}$$

and apply the Goldstone or any equivalent procedure treating $\lambda H_1 - V(\lambda)$ as the perturbation. We then obtain a linked cluster expansion which differs from the spherical one using H_0 as the starting point in three respects. (1) The momentum integrations are carried out above or below the exact Fermi surface. (2) The energy denominators involve the model single-particle energies, $\epsilon(\vec{p})$, which possess the symmetry appropriate to the actual Fermi surface at every density. This assures us, for example, that there are no vanishing energy denominators in the expansion. (3) The diagrams involve not only the two-particle potential, but, also, the model single-particle potential $U(\vec{p})$.

Our result would, of course, be useless if a method were not actually at hand to determine the generalized self-consistent potential $U(\vec{p})$, if not a priori, at least step by step with the energy. The procedure consists in utilization of the defining equation for $\mu = \epsilon(\vec{p}_F)$ and Eq. (5) which together imply that

$$\mu = \epsilon(\vec{p}_F) = p_F^2 + U(\vec{p}_F) \equiv (\partial E/\partial N)_\Omega$$

$$= \frac{d}{d\rho}\left(\frac{E}{\Omega}\right) = \frac{(2\pi)^3}{p_F^2(\omega)} \frac{\delta}{\delta p_F(\omega)}\left(\frac{E}{\Omega}\right). \tag{8}$$

Equation (8) and the linked cluster expansion can be used either iteratively or self-consistently to determine both the energy and the chemical potential. It is important to emphasize that in contrast to previously suggested schemes, exact self-consistency is possible in principle.

It is clear that our method is not confined to the example used, but is applicable to any situation where the symmetry of the ground state is not known to start with but must be determined as part of the solution of the problem.

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INTERACTIONS OF p_σ AND p_π ORBITALS IN TRANSITION ELEMENT FLUORIDES

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We have measured the nuclear magnetic resonance of fluorine in single crystals of $KNiF_3$ and K_2NaCrF_6. Both crystals have high symmetry so that the resonance shifts observed can be interpreted directly in terms of the fraction of unpaired spins in the fluoride ion's $2s$, $2p_\sigma$, and $2p_\pi$ orbitals. The results show relatively large amounts of both p_σ and p_π interactions with the d electrons in these essentially ionic fluorides.

The structure of $KNiF_3$ is cubic perovskite in which the Ni^{++} ions, with site symmetry O_h, are surrounded by a regular octahedron of F^- ions.[1,2] In order to put the Cr^{+++} ion into the center of a regular octahedron of fluorides, we grew a single crystal of K_2NaCrF_6. The structure of this compound has been determined by x-ray diffraction[2]: it is face-centered cubic, $a_0 = 8.266$ A, space group O_h^5-$Fm3m$ with 4 formula units in the unit cell and a Cr-F distance of 1.933 ± 0.011 A. In both crystals the O_h site symmetry makes it possible to identify the one-electron d orbitals with the fluoride ion axes. In $KNiF_3$ the fluorines have site symmetry D_{4h} while in K_2NaCrF_6 they have C_{4v}, so that there are mutually perpendicular directions along which the fluoride ion p orbitals are directed, and in both cases the two sets of p_π orbitals are identical.

The fluorine resonances were measured with a Varian Associates V-4311 induction spectrometer operating at 60.000 Mc/sec. The resonances were displaced from the normal resonance field of ω/γ_N because of dipole fields and hyperfine interactions[3] arising from the unpaired electrons. In Fig. 1(a) we present measurements recently made[4] on a single crystal of $KMnF_3$ oriented so that H_0 can rotate in the (110) plane. The dipole sum, presented as a dashed line, accounts for most of the anisotropic interaction, in contrast to similar measurements on $KNiF_3$ in Fig. 1(b) and K_2NaCrF_6 in Fig. 1(c). In $KNiF_3$ the dipole sum contributes only a small fraction of the observed anisotropy of the shifts while in K_2NaCrF_6 it is out of phase with the observed shifts. The reason for this, and in fact the basic reason for studying the Ni^{++} and Cr^{+++} compounds, is that measurements of the fluorine hyperfine interaction only determine[4] the difference in occupancy of the fluorine p_σ and p_π orbitals. In Mn^{++}, with

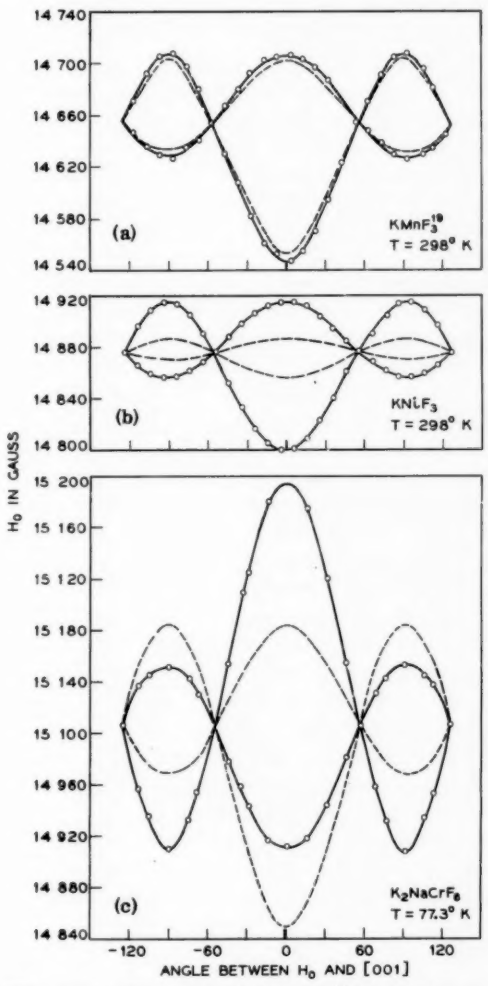

FIG. 1. (a) Resonance field for F^{19} in $KMnF_3$ as a function of angle between H_0 and [001] as H_0 is rotated in (110) plane. The dipole sum, indicated by dashed line, was performed on an IBM 704, and is seen to account for most of the observed anisotropy. For all three compounds the measurements were made at 60.000 Mc/sec; consequently the undisplaced fluorine resonance field should be 14 979.4 gauss. (b) Measurements made under identical conditions on $KNiF_3$. Notice that since the p_π interaction is forbidden the p_σ interaction accounts for most of the observed anisotropy. (c) Similar measurements made at 77.3°K for K_2NaCrF_6. Notice that the measurements are out of phase with the dipole sum as expected for p_π interactions.

a $3d^5$ configuration in a $^6S_{5/2}$ ground state, all five of the one-electron functions are half filled, and these unpaired spins are available to form both σ and π bonds with the ligands. However, Ni^{++} in a $3d^8$ configuration is split by an octahedral crystal field so that the triplet (embodying d_{xy}, d_{xz}, and d_{yz}) lies lower and is completely filled while the doublet (d_{z^2} and $d_{x^2-y^2}$) is a half-filled subshell. Since the triplet mixes with ligand functions to form π bonds and the electrons in the doublet mix to form σ bonds, it is clear that the magnetic electrons in Ni^{++} only mix into the σ bonds. In a complementary fashion Cr^{+++} with a $3d^3$ configuration can only form magnetic π bonds.

The hyperfine interactions of the individual bonds derived from these measurements are expressed by the Hamiltonian[3,5]

$$H = \vec{I} \cdot A \cdot \vec{S}, \qquad (1)$$

in which $\vec{I}$ is the fluorine nuclear spin and $\vec{S}$ the metal ion electron spin. The components of the hyperfine interaction tensor A derived from the experiments are listed in Table I along with f_S and $f_\sigma - f_\pi$, the fraction of unpaired $2s$ and $2p_\sigma - 2p_\pi$ spins in fluoride ion orbitals. The large values of $f_\sigma - f_\pi$ in $KNiF_3$ and K_2NaCrF_6 are noteworthy. The negative value of the latter is caused by $A_\sigma = 0$ to a first approximation so that the large effects of the p_π interactions are not obscured by p_σ interactions of opposite sign. The interactions in $KMnF_3$ and qualitatively similar values reported[3] for MnF_2 in which $f_\sigma - f_\pi = (0.3 \pm 0.3)\%$ are presumably explained by large σ interactions being almost cancelled by the unexpectedly large π interactions. It had been suggested[6] that the s-electron hyperfine interactions could be explained by overlap effects between F^- and Mn^{++} orbitals. Keffer, Oguchi, O'Sullivan, and Yamashita[7] have computed that the p electrons could participate in electron transfer, which is equivalent, in their atomic orbital approach, to covalent bonding in the molecular orbital approach previously used[3,4,8,9] to discuss these results. However, their rough calculation of the p-electron interaction was too small. One possible change, which would improve the agreement with experiment, would be a smaller value for the energy denominator for transfer than the 10-15 ev they estimated. In this event the cubic field splitting between the triplet and doublet levels of 2 ev would be more important and tend to increase the relative amount of p_π bonding. It is clear that the extra degree of freedom accorded the p electrons in a calculation of this sort is required to explain the experimental results.

Table I. The values of A_S and $A_\sigma - A_\pi$ are given in units of 10^{-4} cm^{-1}. Limits of error in $KNiF_3$ and K_2NaCrF_6 are relatively large because of uncertainties in the susceptibilities.

	$KMnF_3$	$KNiF_3$	K_2NaCrF_6
A_S	16.26±0.5	39.1±4	-1.1±0.5
f_S	(0.52±0.02)%	(0.50±0.05)%	-(0.021±0.01)%
$A_\sigma - A_\pi$	0.17±0.1	10.9±1.4	-7.2±1.2
$f_\sigma - f_\pi$	(0.18±0.1)%	(4.95±0.6)%	-(4.90±0.8)%

On the other hand, calculations by Marshall[10] of the overlap effect using expanded Mn^{++} orbitals agreed reasonably well with the relatively small p-electron hyperfine interactions reported for MnF_2. It is clear that this agreement was fortuitous since it depended upon the cancellation of larger p_σ and p_π interactions. Furthermore, the overlap effects alone could not explain the large σ-electron interactions observed in $KNiF_3$ since f_S is the same in these two compounds, and the Ni^{++} ion radial function once adjusted to give the proper s-electron overlap will not give a large enough p-electron interaction.

The small negative isotropic shift in K_2NaCrF_6 can be explained[11,12] by excited states and its magnitude used to determine their contribution to the eigenstate of the system. The effects of these large p_σ and p_π interactions upon crystal field splittings, spin-orbit interactions, term values, and cation interactions are being studied.

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FERMI SURFACES OF GOLD AND SILVER FROM ULTRASONIC ATTENUATION*

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Recent experiments have confirmed that the Fermi surface in copper contacts the Brillouin zone boundary in the [111] direction as suggested by Pippard.[1-5] Using the low-temperature magnetic dependence of ultrasonic attenuation, we have found that the Fermi surfaces in gold and silver contact the zone boundary in a similar way. In gold the size of the contact area and the shape of the "neck" joining this to the main body of the surface have been obtained in a very direct way.

An oscillatory variation of ultrasonic attenuation in a magnetic field occurs because of geometrical coincidences between certain electron orbits and the periodic electric fields accompanying the sound wave. Liquid helium temperatures are needed to have the mean free path longer than the wavelength, λ. The effect, when $\vec{H}\perp\vec{q}$ (where $\vec{q}$ is the propagation vector of the wave), gives the dimensions and ultimately the shape of the Fermi surface because the period observed in $(H\lambda)^{-1}$ is determined by the Fermi momentum perpendicular to $\vec{H}$ and $\vec{q}$ at those parts of the surface where this momentum is extremal. If p is such a momentum, and P is the corresponding period in $(H\lambda)^{-1}$, then $p=e/(2cP)$.[6,7]

Magnetic field oscillations were observed in single crystals of zone-refined gold and silver at frequencies up to 154 Mc/sec at a temperature of 4.2°K. Longitudinal waves were propagated along both [001] and [011], and the direction of $\vec{H}$ varied while keeping $\vec{H}\perp\vec{q}$. Stronger oscillations were observed in gold than in silver, as many as 19 maxima and minima being found in certain orientations.

If the Fermi surface contacts a zone boundary, there will be a minimal cross section at the boundary; thus, if touching occurs in [111], there should be a long-period oscillation due to electrons circulating around this "neck" when $\vec{H}$ is along [111] and $\vec{q}$ is along [011]. Such a period has been observed in copper.[3] A pronounced period, which is shown in Fig. 1(a), is found also in gold and is the strongest evidence for the surface making contact. The momentum calculated from this period is 0.22, in units of 10^{-19} g cm sec^{-1}. (In the remainder of the paper all momenta will be expressed in these units.) This

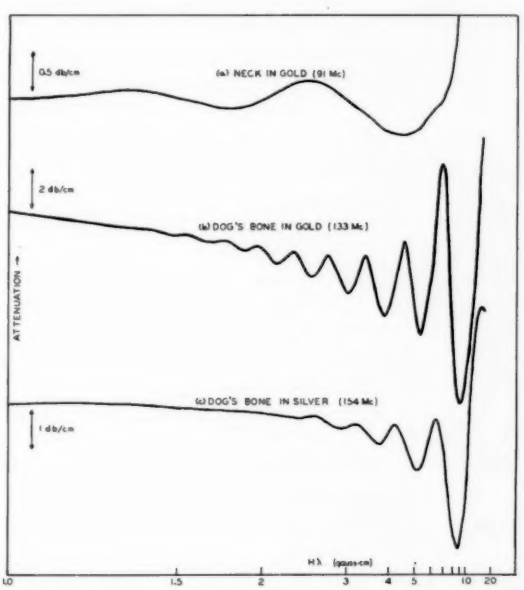

FIG. 1. Relative attenuation as a function of $H\lambda$ plotted on a reciprocal scale. (a) Attenuation in gold with $\vec{q}$ along [011] and $\vec{H}$ along [111]. (b) Attenuation in gold with $\vec{q}$ along [001] and $\vec{H}$ along [110]. (c) Attenuation in silver with $\vec{q}$ along [001] and $\vec{H}$ along [110]. (Note that attenuation scales are different for each curve.)

should be the radius of the touching area on the [111] zone boundary, which is 1.41 units away from the center of the zone. When $\vec{H}$ is moved away from [111], the neck period becomes shorter corresponding to the widening of the neck as it joins the main part of the surface. This angular dependence gives the detailed shape of the neck, the result being shown in Fig. 2.

The strongest period in gold appears with $\vec{q} \parallel [001]$ and $\vec{H} \parallel [110]$, as shown in Fig. 1(b); the corresponding momentum is 1.09 units. This period is interpreted as follows: Because contact occurs, the Fermi surface can be considered as a body-centered array of spheres (approximately) joined along [111], and corresponding directions, by the neck shown in Fig. 2; the central extremal cross section is a "dog's bone" shaped orbit when the field is along [110]. This is a hole-type motion involving four zone boundary reflections. The momentum of 1.09 units, therefore, is associated with half the length of this dog's bone. Since the distance from the center of the dog's bone to the center of the zone is 2.30 units, this period implies that the radius of the main surface along [110] is 1.21 units.[8]

For $\vec{q} \parallel [001]$ and $\vec{H}$ near [100], there is a much weaker period undoubtably due to electron orbits about the central body of the surface. From this period the radius of the main body along [100] is estimated as 1.26 units. The angular variation of this and the dog's bone period can be used to establish the shape of a principal cross section of the main body of the Fermi surface, the values of 1.26 units along [100] and 1.21 units along [110] being the maximum and minimum dimensions. If the surface in gold were a sphere that contained one electron per atom, its radius would be 1.27 units.

Even though the original silver was nominally purer than the gold, the oscillations for a given frequency were much weaker. The dog's bone orbit gave the most distinct period, this being shown in Fig. 1(c) for a frequency of 154 Mc/sec. A momentum of 1.05 units is found, which implies a main body radius of 1.25 units along [110] for silver. The zone and the ideal Fermi sphere are almost identical in silver and gold. There was an indication of a very weak neck period in silver, enough to give a rough estimate of 0.18 unit for the radius of contact. Even though the neck period is not clear, the strong dog's bone period, when $\vec{H} \parallel [110]$, leaves little doubt that the Fermi surface of silver makes zone boundary contact in a way similar to the other noble metals.

Our results agree with very recent de Haas–van Alphen measurements of Shoenberg.[9] He has observed the dog's bone oscillation in gold (the name is his) and the neck period in all three metals. The two methods are complementary in that the de Haas–van Alphen effect determines the area of an orbit, while the ultrasonic effect determines its length. Moreover, it seems that orbits involving zone boundary reflections give stronger oscillations than the main surface orbits with the ultrasonic effect, but that the opposite is true for the de Haas–van Alphen effect.

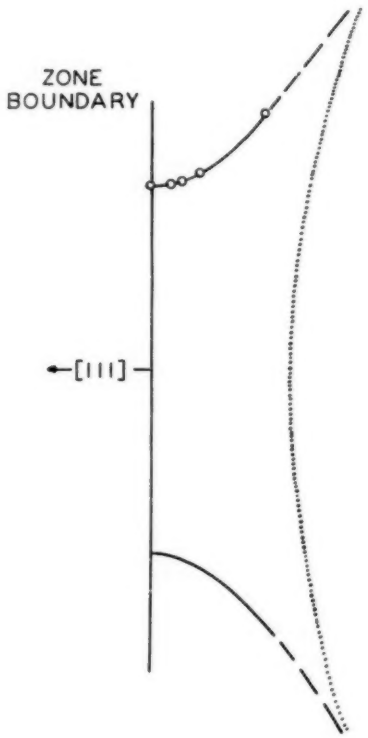

FIG. 2. Inferred shape of the touching "neck" in gold at the [111] zone boundary. The measured radius at the boundary is 0.22×10^{-19} g cm sec^{-1}. The dotted curve is a circle of radius 1.25×10^{-19} g cm sec^{-1} drawn from the center of the zone.

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EFFECTS OF POLYTYPISM ON THE ELECTRON PARAMAGNETIC RESONANCE OF $K_3Co(CN)_6$ AND OTHER SPECTROSCOPIC IMPLICATIONS*

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The electron paramagnetic resonance (EPR) spectra of Cr-doped $K_3Co(CN)_6$ crystals show unexpected splittings.[1] In every crystal investigated, at least four prominent lines, occurring in sets of two, were detected for each $\Delta M = \pm 1$ Cr^{+++} (in Co^{+++} sites) transition. In several crystals, at least two additional pair-wise equivalent resonances were noticed. The resonance geometry for the prominent lines is indicated in Fig. 1; the room temperature spectral data obtained at a frequency of 9.3 kMc/sec are given in Table I.

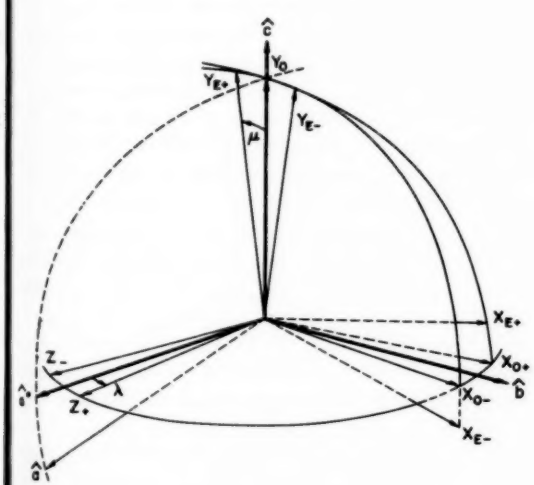

FIG. 1. Geometry of the four prominent Cr^{+++} resonances in polytypic K_3Cr-$Co(CN)_6$.

The existence of different well-defined stacking periodicities in crystals of a given chemical species is termed polytypism. Each layering (stacking) sequence corresponds to a specific polytype and generates a crystal (or region in a crystal) which is singularly described by a given unit cell and crystal structure. In general, several distinct polytypes may coexist in a visually homogeneous crystal.

Recently, pure and doped (0.1% Cr) crystals of $K_3Co(CN)_6$ were found to be polytypic by single-crystal x-ray diffraction.[2] The unit cells of the various polytypes are derived by rotation of (100) one-layer monoclinic lamellae about c and their stacking on (100). The geometry of the two most prevalent polytypes, $1M$ (one-layer monoclinic) and $2Or$ (two-layer orthorhombic) is illustrated

Table I. Room temperature spectral data at 9.3 kMc/sec.

Ordinary lines	Extraordinary lines
$\lambda = \pm 6.5°$	$\lambda = 6.5°$, $\mu = 8.25°$
$\mu = 0$	$\lambda = -6.5°$, $\mu = -8.25°$
$D = 0.07832$ cm^{-1}	$D = 0.07866$ cm^{-1}
$E = 0.00902_6$ cm^{-1}	$E = 0.00973_0$ cm^{-1}
$g_y = 1.991_2$	$g_y = 1.992_3$
$g_x = 1.993_6$	$g_x \simeq 1.993_3$

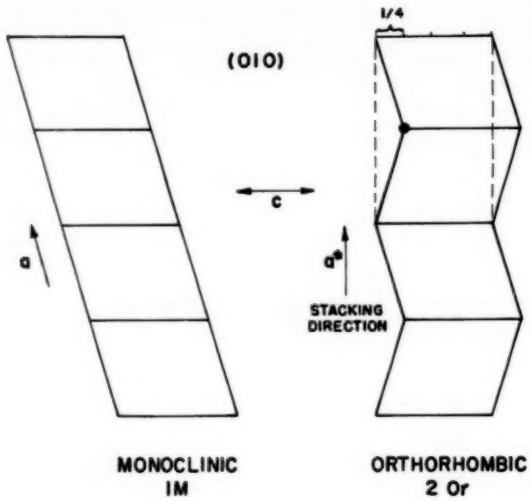

FIG. 2. Relation of the 1*M* to the 2*Or* cell as seen on (010). [$Co(CN)_6^{---}$ octahedra are centered at the stacking interfaces.]

in Fig. 2. The unit cell data are given in Table II.

Upon correlation of the microwave and crystallographic data, one prominent pair of lines (termed Ordinary because these are the only ones listed by Baker et al.[3] was found to correspond to the 2*Or* polytype, the other prominent pair (the Extraordinary lines) to the 1*M* polytype. (The *O*-lines were incorrectly associated with the 1*M* structure in reference 3.) Thus, both the 2*Or* and 1*M* polytypes each have only two nonequivalent Co sites.

In addition to these 1*M* and 2*Or* polytypes, structures having larger unit cells (e.g., 8*M*) were shown by x-ray diffraction. Previous work[2] had demonstrated 3*M* and 7*M* polytypes. Correlations between the EPR and x-ray diffraction results indicate that these additional structures seem to contribute to the *O* and *E* resonances as well as to the fifth and sixth (or more) resonanc

Since the individual polytypes are generated by 180° flips of the 1*M* elementary unit cell and subsequent stacking, the octahedral environment surrounding the Co sites would be unchanged if the intrinsic structural parameters were invariant. If the resonances were affected only by nearest neighbor ions, then no departure from the *E* spectra (1*M*) would be anticipated from a microwave point of view. The existence of both *O* and *E* spectra thus demands that Cr^{+++} (in Co^{+++} sites) senses the 1*M* and 2*Or* lattice geom etries differently—either due to distortions of the $(CN)_6$ octahedron, or to the influence of ions located outside the octahedron, or both. These results, together with other considerations, suggest that the Cr^{+++} "sphere of influence" extend at least over a single "1*M*" elementary cell volume.

This spectral polytype effect is not uniquely associated with the presence of Cr^{+++} as a lattice substituent in $K_3Co(CN)_6$. As many as eight sets of lines have been observed in the EPR of Fe^{+++}-doped $K_3Co(CN)_6$.[4] Furthermore, a fourfold splitting of the quadrupole structure of the Co^{59} nuclear magnetic resonance (NMR) has recently been observed in $K_3Co(CN)_6$.[5] Although detailed data are not available, it seems clear that this is the polytype effect as seen in the NMR frequency domain. It might be postulated that these polytype effects could also be noticed in other portions of the electromagnetic spectrum. Splittings or fine structure of the Cr^{+++} absorption spectra, for example, might be observable in the optical region. Similarly, the detailed interpretation of magnetic susceptibility measurements might be complicated by these polytypic effects.

Polytypism provides a logical explanation for some unexpected "hole-burning" effects and linewidth vs concentration results recently found in the EPR of Cr^{+++}-doped $K_3Co(CN)_6$.[6] As indicated in reference 1, the *O* and *E* lines generally are not coincident in frequency for an arbitrary crystal direction and applied magnetic field value. If the resonances are not resolved, the resultant smeared line is expected to be spatially inhomogeneous in the same sense that frequently obtains in NMR. Application of these considerations to the results of reference 6, without any

Table II. The unit cell data.

	2*Or*	1*M*
a (A)	13.31 ± 0.04	7.00 ± 0.02
b (A)	10.37 ± 0.02	10.38 ± 0.01
c (A)	8.37 ± 0.01	8.37 ± 0.02
β	90°	107°19′ ± 3′
Molecules/unit cell	4	2
Space group	*Pnca*	$P2_1/c$

additional ad hoc suppositions, follows at once.

Resonance studies on polytypic structures provide a means of exposing the subtler aspects of the crystal field concept to critical examination. In Bleaney and O'Brien,[7] for example, certain expressions relating D and E to the g_x, g_y, g_z values are given on the basis of spin Hamiltonian theory. However, a self-consistent numerical formulation for the Cr^{+++} lines was not possible either with the experimental values of reference 3 or with our values. It has been suggested, in the case of reference 3, that the precision of the data was insufficient to substantiate the above criticism.[8] Nevertheless, the possibility remains that a new approach to the spin Hamiltonian is desirable.[9] (Additional internal inconsistencies of order $1:10^3$ arise in fitting the data along two principal axes to the spin Hamiltonian.) Optical results particularly would be of great assistance in resolving such problems. These matters are under current consideration.

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QUADRUPOLE MOMENT OF Fe^{57m}†

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Fe^{57m} ($I=\frac{3}{2}$) decays by an $M1$ transition to Fe^{57} ($I=\frac{1}{2}$) with emission of a 14.4-kev γ ray. Recently[1] the quadrupole coupling constant, e^2qQ, of Fe^{57m} in α-Fe_2O_3 has been found to be $(-23.0\pm5.8)\times10^{-9}$ ev ($=-5.6\pm1.4$ Mc/sec). The absolute value of the quadrupole coupling constant of Al^{27} in α-Al_2O_3 is[2] 2.393 Mc/sec. We will show how a value of the quadrupole moment of Fe^{57m} can be deduced from these data using a model of the ionic lattices.

We suppose that the isomorphic lattices of α-Al_2O_3 (corundum) and α-Fe_2O_3 (hematite) consist of tripositive metal ions and negative $O^=$ ions. If this be true the field gradient acting on the metal ion can be put in the form cea^{-3}, where c is a constant depending on the unit cell parameters (α, u, v) and a is the unit cell edge. The various crystallographic parameters[3] are summarized in Table I. These parameters are so similar that we are justified in assuming that the constant c is the same for both lattices. c is formally defined by the summation

$$\frac{1}{e}\sum_i (ez_i)\frac{(3Z_i^2-R_i^2)}{R_i^5}, \qquad (1)$$

where the sum is over all ions in the crystal. The ith ion has a charge z_ie and coordinates

Table I. Crystallographic data on α-Al_2O_3 and α-Fe_2O_3.

	a (A)	α	u	v	e^2qQ/h (Mc/sec)	Resonant nucleus
α-Al_2O_3	5.13	55°6′	0.105	0.303	±2.393	Al^{27}
α-Fe_2O_3	5.4135	55°17′	0.105	0.292	-5.6	Fe^{57m}

X_i, Y_i, Z_i with respect to the given nucleus. The Z axis is the threefold axis of the crystal on which the aluminum atoms lie. For the α-Al_2O_3 lattice the series has been summed[4] and the value of c is 10.421, but the sign of c is more important here than its precise magnitude.

The field gradient seen by the nucleus is not the same as the field gradient seen by the ion but must be corrected by a so-called antishielding factor $(1-\gamma_\infty)$ which takes into account the polarization of the electron shells on the given ion. Now the factor $1-\gamma_\infty$ has not been calculated for Fe^{+++} but may be interpolated from results[5,6] obtained for Al^{+++} and Ga^{+++} for which γ_∞ is -2.59 and -8.75, respectively. As the ions differ by the electrons $3s^23p^63d^{10}$, γ_∞ for Fe^{+++} must be reasonably close to that for Ga^{+++} so that one can guess at the value -6.5 ± 1.0. (This value is obtained by subtracting half the $3d$ contribution from the Ga^{+++} value.)

The quadrupole moment of Fe^{57m} can then be determined from the equation[4]

$$(e^2qQ)_{\text{measured}}=(1-\gamma_\infty)ce^2a^{-3}Q, \qquad (2)$$

which will be valid if our simple picture holds of the lattice as an array of nonoverlapping ions. Polarization of the ions has been neglected except for the electron shells surrounding the given ion. This is a rather oversimplified model of the crystal. If it is used for Al_2O_3, a value of Q for Al^{27} of 0.247 barn is obtained as compared to the atomic beam value of 0.149 barn. Using this equation for Fe^{57m}, one obtains a value for Q of -0.33 barn.

A more reliable determination of Q_{57m} would use Eq. (2) twice, once for Al^{27} and once for Fe^{57m}, and eliminate the constant c. In other words, aside from the factors a^{-3}, we are assuming that Fe^{+++} in Fe_2O_3 sees the same field gradient as Al^{+++} in Al_2O_3. Yet admittedly we cannot calculate this field gradient with any particular precision. Instead we use the Al_2O_3 measurement to determine the field gradient for us, and we write

$$\frac{(e^2qQ)_{57m}}{(e^2qQ)_{27}}=\left(\frac{a(Al_2O_3)}{a(Fe_2O_3)}\right)^3\frac{(1-\gamma_\infty)_{Fe^{+++}}}{(1-\gamma_\infty)_{Al^{+++}}}\frac{Q_{57m}}{Q_{27}}, \qquad (3)$$

from which one obtains $Q_{57m}=-0.19$ barn. Here we have made the additional assumption, justified by the approximate validity of Eq. (2), that the coupling constant of Al is positive in α-Al_2O_3.

The uncertainty in the quadrupole moment just computed is difficult to estimate. The factor γ_∞ was calculated for Al^{+++} using Hartree functions and not Hartree-Fock functions; γ_∞ of Fe^{+++} was only crudely estimated and a calculation is possible with the wave functions obtained by Watson.[7] On the experimental side, the assumption that the field gradients in the two lattices are essentially the same could be tested by studying a solid solution of $(Fe^{57})_2O_3$ in Al_2O_3. Other isomorphous lattice pairs might also be studied.

Our conclusion is that the quadrupole moment of Fe^{57m} is large and negative but that the numerical value quoted is very rough. The individual-particle shell model does not make an unambiguous prediction for this odd-neutron nucleus.

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PRODUCTION OF NUCLEON ISOBARS IN PROTON-PROTON COLLISIONS*

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The isobar model of pion production has developed from the work of Watson,[1] Peaslee,[2] Lindenbaum and Sternheimer,[3] and Mandelstam,[4] and has been supported by experimental work in both nucleon-nucleon[5-9] and pion-proton[10,11] collisions. In this model it has been assumed that the process of meson production proceeds through two stages, first the production of an isobar (a nucleon in an excited state) and secondly the decay of the isobar to a normal nucleon and a π meson. Calculations have been made[3] on the basis of this model by assuming that the probability for producing an isobar of given mass can be related to the variation of cross section with energy for the (3/2, 3/2) resonant state of the π-p system. Because of the finite width of this resonance there is a distribution of possible values of the isobar mass with a maximum probability at 1.23 Bev, given by the resonant pion kinetic energy of 195 Mev, and leading in p-p collisions to a corresponding maximum in the energy spectrum of the protons recoiling in the production process. Experimentally this peak should be superimposed upon a fairly smooth distribution of energies representing decays of the isobar by the proton mode and should contain 3/14 of all the protons observed.

We have investigated the validity of the isobar model by observing the momentum spectrum of inelastically scattered protons from p-p collisions with the arrangement shown in Fig. 1. The external proton beam from the Cosmotron was focussed at the center of a 15-in. long liquid hydrogen target. The momentum spectrum of protons emerging at 4.8° to the incident beam was determined using an analyzing magnet and a counter telescope, giving a momentum resolution of ±3.4%. It was unnecessary to discriminate against pions since these were kinematically excluded from the momentum region of interest. The incident proton beam was monitored by a thin argon-filled ionization chamber which was placed in front of the target.

Incident proton energies of 1.04, 1.20, 1.35,

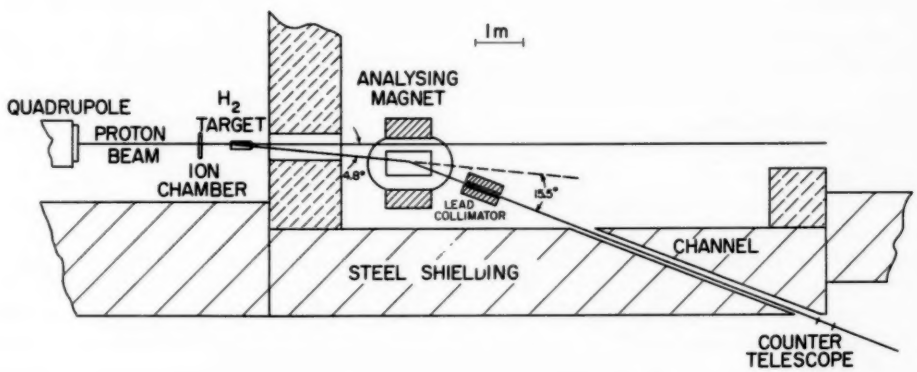

FIG. 1. Experimental arrangement.

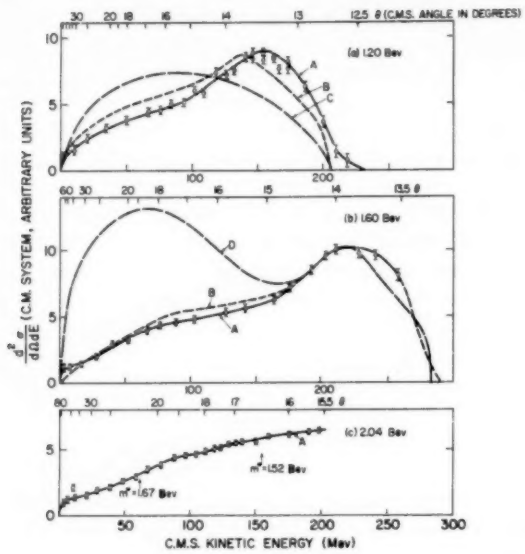

FIG. 2. Center-of-mass energy spectra of inelastically scattered protons from p-p collisions at (a) 1.20, (b) 1.60, and (c) 2.04 Bev. Curves A fit the experimental points, curves B are spectra expected for single meson production from the isobar model, curve C is the three-body phase space factor, and curve D is the predicted spectrum from the isobar model for one- and two-meson production in the ratio of their observed production cross sections. The center-of-mass angle of observation is shown above each section.

1.60, 1.74, and 2.04 Bev were used. The spectra of inelastically scattered protons, transformed into the center-of-mass system for three of these energies, are shown in Fig. 2. At the first two energies mentioned above, the elastically scattered protons were within the momentum range of the spectrometer but these are not shown in the figure. The statistical error in the number of counts was less than $\pm 1\%$ over most of the momentum range covered; the errors shown represent an estimate of beam monitor fluctuations. The background counting rate, obtained from an empty target, was almost constant except at momenta close to that of the elastically scattered protons and equal to about 17% of the counting rate obtained from a full target in the region of the maximum in the inelastic scattering.

The theoretical curves B and D in Fig. 2(a) and 2(b) were calculated by Sternheimer[12] assuming an isotropic distribution for the production and decay of the isobar but without folding in the experimental resolution. The agreement between the experimental curve A and the calculated curve B at 1.20 Bev [Fig. 2(a)] is sufficiently good to give strong support to the isobar model. There is no agreement with the distribution predicted on a statistical basis (curve C). The peak in curve B indicates an isobar mass of 1.19 Bev which, when compared with the value 1.23 Bev obtained from π-p scattering data, is a bit outside our estimated experimental uncertainty. Similar shifts are observed at 1.04, 1.35, and 1.60 Bev incident energies, giving masses of 1.19, 1.18, and 1.21 Bev, respectively.

At 1.6 Bev [Fig. 2(b)] the experimental spectrum A agrees well with the theoretical curve B, calculated for single meson production only, which is surprising because the known ratio for double to single meson production is 0.5 ± 0.3 at this energy[8] so that protons from the double production process should add to the spectrum to give curve D. This discrepancy seems to imply a radical difference in the angular distributions of the two processes so that the single production recoils predominate at smaller angles. It is possible that the introduction of appropriate angular distributions into the theory would account for some of the disagreement shown in Fig. 2(a), but probably would not explain the shift in the peak.

In addition to the 3/2, 3/2 resonance so clearly evident in our data, previous work on pion photoproduction[13,14] and pion-nucleon scattering[15,16] has indicated the existence of additional resonances. The first two belong to the $T = 1/2$ system and correspond to total masses of 1.52 and 1.67 Bev. If these new states are interpreted as isobars they should be formed in p-p collisions; in fact the Clebsch-Gordan coefficients for the production of "recoil" protons together with $T = 1/2$ isobars give them a statistical weight relative to "decay" protons of 3, as compared with 3/11 for the $T = 3/2$ isobar case.

The expected positions of peaks in the proton spectrum for an incident energy 2.04 Bev are shown by arrows in Fig. 2(c), which gives the experimental results at this energy. There is no evidence for production of these isobars, either at this or at the lower energies investigated.

It is proposed to observe these spectra at several more angles and higher incident energies in order to investigate the angular distribution of the isobar production and to continue searching for the formation of higher mass isobars.

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NEUTRINOS EMITTED IN β DECAY AND μ CAPTURE*

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Lee and Yang have recently drawn attention to the possibility that the neutrinos ν_1, ν_2, emitted in μ^- capture and β^+ decay, respectively,

$$\mu^- + p \rightarrow n + \nu_1,$$
$$(A,Z) \rightarrow (A,Z-1) + e^+ + \nu_2, \qquad (1)$$

may not be identical particles.[1] Indeed, if the absence of the photodecay $\mu \rightarrow e + \gamma$ is to be reconciled with the intermediate vector boson theory[2] of weak interactions, then it must be assumed that:

(a) the helicity of ν_1 is the same as the helicity of ν_2;

(b) ν_1 and ν_2 must differ in some internal coordinate which may, perhaps, be the analog for leptons of the "strangeness" of hyperons and K mesons.

After the discussion by Lee and Yang had appeared, it was reported that an attempt to measure the helicity of the μ^- emitted in π^- decay had yielded no definite result.[3] Therefore, since the helicity of ν_1 is still an open question, it is of some interest to examine the consequences of an assumption alternative to (a), namely:

(a′) ν_1 and ν_2 are created with opposite helicities in μ^- capture and β^+ decay, respectively.

Let us suppose that the coupling of leptons to bare nucleons is V-A, both for β decay and μ capture.[4] Since the helicity of ν_2 in (1) is negative,[5] the interaction Hamiltonian is

$$\mathcal{H}_{\pm} = (\bar{\psi}_p \gamma_\lambda (g_V + g_A \gamma_5) \psi_n)[(\bar{\psi}_\mu \gamma_\lambda (1 \pm \gamma_5) \psi_{\nu_1}) + (\bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_{\nu_2})] + \text{H.c.}, \qquad (2)$$

where the positive sign corresponds to assumption (a) and the negative to (a′). From the conservation of spin and momentum in π decay,

$$\pi^- \rightarrow \begin{cases} \mu^- + \bar{\nu}_1 \\ e^- + \bar{\nu}_2 \end{cases} \qquad (3)$$

(for clarity we shall only consider the π^-), it follows that $\mathcal{H}_-$ implies the same branching ratio of the $(\pi - e)$ mode to the $(\pi - \mu)$ mode as does $\mathcal{H}_+$; this theoretical ratio is in agreement with the experimental value.

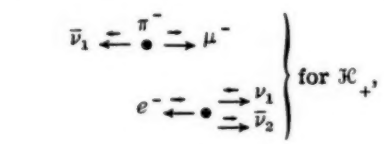
Consider now the $\pi - \mu - e$ chain: the only established experimental facts are[5]:

(*i*) the electrons $e^{\pm}$ are emitted predominantly antiparallel to the $\mu^{\pm}$ momentum;

(*ii*) the helicity of $e^{\pm}$ is ± 1 (the electron mass is neglected);

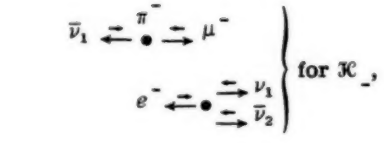
(*iii*) the measured values of the Michel parameters in μ decay are $\rho \approx 0.75$, $\delta \approx 0.6$, $|\xi| \approx 0.8 - 1.0$.

When the electron has its maximum momentum, the helicity assignments consistent with (*i*), (*ii*), and the conservation of spin and momentum are



for $\mathcal{H}_+$, (4)

and



for $\mathcal{H}_-$, (5)

where the short arrow in each case denotes the direction of the spin. (The designation of the neutrinos in μ decay will be discussed later.) The configuration in (4) leads to the usual V,A Hamiltonian for μ decay, while that in (5) leads to S, T, P coupling:

$$\mathcal{H}_-(\mu - e + \nu_1 + \bar{\nu}_2) = \sum_{i=S,T,P} g_i(\bar{\psi}_e \Gamma_i(1-\gamma_5)\psi_\mu)(\bar{\psi}_{\nu_1}\Gamma_i(1-\gamma_5)\psi_{\nu_2}) + \text{H.c.}, \quad (6)$$

$$\Gamma_S = 1, \quad \Gamma_P = \gamma_5, \quad \Gamma_T = \frac{i}{2\sqrt{2}}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu).$$

The electron spectrum which is obtained from $\mathcal{H}_-(\mu - e + \nu_1 + \bar{\nu}_2)$ is

$$dN_-(x,\theta) \sim \{3(1-x) + 2\rho(\tfrac{4}{3}x - 1) + \xi\cos\theta[1 - x + 2\delta(\tfrac{4}{3}x - 1)]\}x^2 dx d\Omega, \quad (7)$$

where $x \equiv$ (electron energy/maximum electron energy), $\theta \equiv$ angle between electron momentum and muon spin direction, and

$$\rho = 12|g_T|^2/(|g_S + g_P|^2 + 12|g_T|^2),$$

$$\xi = (28|g_T|^2 - 3|g_S + g_P|^2)/(|g_S + g_P|^2 + 12|g_T|^2),$$

$$\delta = \rho/\xi.$$

When $g_S = g_P = \pm g_T$, then

$$\rho = \delta = 3/4, \quad \xi = 1, \quad (8)$$

which values are consistent with the data in (*iii*).

It is assumed that the neutrinos emitted in μ^- decay are ν_1 and $\bar{\nu}_2$ in order to conserve lepton charge and any other such quantum number as may be associated with leptons [see assumption (b)]. The assumption (a′) clearly implies that ν_1 and ν_2 are both four-component neutrinos since their respective helicities in μ decay [see (5)] are opposite from those in β decay and μ capture. While it does not matter, in (4), whether ν_1 and $\bar{\nu}_2$ are identical particles, the Pauli principle requires that they be distinct in configuration (5), i.e.,

$$\nu_1 \neq \bar{\nu}_2. \quad (9)$$

It is now clear that the immediate consequence of assumption (a′) must be the rejection of (*i*) the universality of V-A coupling in weak interactions (*ii*) the "two-component" neutrino, and (*iii*) the intermediate vector boson theory, at least in its present form. A further consequence is that the neutrino-capture reaction

$$\pi^+ - \mu^+ + \nu_1, \quad \nu_1 + n - p + e^-, \quad (10)$$

and the second-order process[7]

$$\mu^- + p - n + \nu_1 - e^- + p \quad (11)$$

do not occur, whether ν_1 is identical to ν_2 [which would be allowed by (9) if ν_2 is a Dirac particle] or not; for, according to $\mathcal{H}_-$ [see (2)], the helicity of ν_1 is opposite to that required for absorption by a neutron together with the simultaneous emission of an electron.

The validity of (a′) depends, of course, upon the helicity of the muon emitted in $(\pi - \mu)$ decay. We should like to draw attention to the consequences described above, in order to encourage attempts at this difficult experiment.

The author would like to thank W. Holladay, M. E. Ebel, and M. H. Good for their encouragement and helpful comments.

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*Supported by the U. S. Atomic Energy Commission.

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[4]M. L. Goldberger and S. B. Treiman, Phys. Rev.

111, 354 (1958).

[5]For a complete list of references on weak interactions see the review articles of E. J. Konopinski, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99; and J. Sakurai, Progress in Nuclear Physics (Butterworths-Springer, London, 1959), Vol. 7, p. 244.

[6]Starting from a somewhat different point of view, L. B. Okun and V. M. Shekhter, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1250 (1958)[translation: Soviet Phys.-JETP 34(7), 864 (1958)], obtain the same result as in this paragraph. The author is indebted to Professor R. G. Sachs for this reference.

[7]If the intermediate boson does not exist, (11) is the only possible mechanism for the decay $\mu+p\rightarrow n+e$: see S. P. Rosen, Nuovo cimento 15, 7 (1960).

EFFECT OF THE PION-PION RESONANCE ON K^--p SCATTERING*

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Considerable theoretical interest has been shown in the interpretation of the low-energy K^--p scattering data. The major effort has been directed toward a convenient parameterization that takes into account the kinematic features of the many competing channels ($K^-+p-\overline{K}^0+n, \pi Y$) and the "standard" Coulomb and mass-difference corrections.[1-4] In attempts to draw conclusions about the nature of the basic K-meson nucleon interaction, the assumption has been made that $k\cot\delta$ is essentially constant and equal to the reciprocal (complex) scattering length. We have examined the validity of this approximation on the basis of the Chew-Mandelstam program.[5] We found that the two-pion exchange, which determines the long-range tail of the K-N interaction, gives a substantial energy dependence to $k\cot\delta$.

A qualitative determination that remains to be made is the sign of the real part of each ($I=0, 1$) scattering length.[6] Two attempts to do this have been based on properties of the Coulomb-nuclear interference. The angular distribution at 172 Mev/c (laboratory-system momentum) favors a constructive interference[7] and implies a positive sign. The other attempt is based on the apparent leveling off and decrease of the elastic-scattering cross section with decreasing momentum ($P_L<150$ Mev/c). Jackson and Wyld have suggested that this behavior is due to destructive interference and consequently concluded that the sign is negative.[3] The energy dependence arising from the two-pion exchange provides an alternative interpretation of the leveling off. This can be seen qualitatively from the following argument: by virtue of the small pion mass (compared with the K-meson mass) the two-pion exchange determines the longest range part of the K^--p potential. This suggests that we regard the nuclear interaction as made up of the two-pion contribution plus another part of shorter range, representing the net effect of everything else. The peculiar energy dependence is interpreted as the destructive interference between the two parts. Knowledge of the sign of the two-pion contribution then leads to a determination of the sign of the scattering length. Recent advances in the theory of the pion-pion interaction[8] and of the electromagnetic structure of the nucleon[9] make it possible to calculate the sign and estimate the magnitude of the two-pion contribution.

From their theoretical study of the nucleon electromagnetic structure, Frazer and Fulco[9] have inferred a resonance in the $I=1$, $J=1$ state of pion-pion scattering. Quantitative conclusions are still uncertain,[10] but it seems likely that the two-pion contribution accounts for a large fraction of the isotopic vector charge and magnetic moment. On the basis of their results, we suggest that the charge structure of the K meson also receives a sizable contribution from the two-pion state. The consistency of this hypothesis and its consequences for πK scattering have been studied.[11] This hypothesis provides an estimate of the order of magnitude of the matrix element for emission of two pions by a K meson ($\pi\pi|K\overline{K}$). Using this estimate, we have calculated the contribution of the two-pion exchange to the $\overline{K}$-nucleon interaction. (The details of this calculation will be presented in a separate report.)

We denote the S-wave $\overline{K}$-nucleon elastic-scattering amplitude by g. It is an analytic function in the cut s-plane (s is the square of the total energy in the $\overline{K}$-N center-of-mass system). The physical branch cut starts at $s=s_0=(M_N+m_K)^2$ and

extends to $+\infty$. There are other "right-hand" cuts starting from the thresholds of the dynamically coupled channels, $(m_\pi + M_\Lambda)^2$ and $(m_\pi + M_\Sigma)^2$.[12] The unitarity condition, which establishes certain nonlinear relations between the amplitudes g_{ij} of various coupled processes, is expressed by

$$\mathrm{Im}(g^{-1})_{ij} = -\frac{k_i \theta_i \delta_{ij}}{s^{1/2}(E_i + M_i)}, \qquad (1)$$

$$\theta_i = 1 \quad \text{if} \quad W_i < s^{1/2},$$

$$= 0 \quad \text{if} \quad W_i > s^{1/2},$$

where k_i, E_i, and W_i are the center-of-mass momentum, baryon energy, and threshold energy of the ith channel. The dynamical singularity arising from the two-pion exchange is a branch cut extending from the left up to the point $s = s_2 = [(M_N^2 - m_\pi^2)^{1/2} + (m_K^2 - m_\pi^2)^{1/2}]$. Relative to the other dynamical singularities, this cut is very close to the physical region $s \geq s_0$, and should produce the strongest energy dependence in $k \cot\delta$.

We denote the elastic $\bar{K}N$ amplitude in the state of isotopic spin I by g_I, where $I = 0, 1$. These two amplitudes are related by crossing symmetry to amplitudes $g^{(+)}$ and $g^{(-)}$ that depend on the isotopic spin $I' = 0, 1$ of the exchanged pion pair, namely

$$g^0 = g^{(+)} - 3g^{(-)},$$

$$g^1 = g^{(+)} + g^{(-)}. \qquad (2)$$

The magnitude of the discontinuity across the two-pion cut is expressed for our purpose as

$$\int_{s_1}^{s_2} ds\, \mathrm{Im} g^{(-)}(s) = R_1 \approx 4 M_N^4 \text{ fermi}, \qquad (3)$$

where s_1 is approximately equal to $80\, m_\pi^2$. Assuming that only the resonant $g^{(-)}$ amplitude is important, we have three specific predictions: (a) the isotopic spin ratio R_0/R_1 is -3, (b) the sign of R_1 is positive, and (c) the expected magnitude of R_1 is rather large.

The normalization of our amplitude g is given by its relation to the $\bar{K}$-N elastic scattering phase shift,

$$g(s) = s^{1/2}(E + M)/(k \cot\delta - ik). \qquad (4)$$

In this note we use only a very simple model in which the effect of the two-pion cut is represented by a δ function at the position $s = a = 93\, m_\pi^2$ in the amplitude g. The dynamical singularities in all other channels are represented by subtraction constants.

This model, combined with the many-channel unitarity condition, Eq. (1), leads to the following expression for $k \cot\delta$:

$$k \cot\delta_I = \left(l(s) + \frac{1}{f_I(s) + z_I} \right) s^{1/2}(E + M), \qquad (5)$$

where

$$l(s) = \frac{s - s_0}{\pi} P \int_{s_0}^{\infty} \frac{ds'\, \mathrm{Im}(g^{-1})_{11}}{(s' - s)(s' - s_0)} - \frac{a - s_0}{\pi} \int_{s_0}^{\infty} \frac{ds'\, \mathrm{Im}(g^{-1})_{11}}{(s' - s_0)(s' - a)},$$

$$f_I(s) = \frac{1}{\pi} \left(\frac{1}{s - a} - \frac{1}{s_0 - a} \right) \frac{R_I}{1 + \beta R_I},$$

$$\beta = \frac{1}{\pi^2} \int_{s_0}^{\infty} \frac{ds'\, \mathrm{Im}(g^{-1})_{11}}{(s' - a)^2},$$

and z_I is a complex quantity which depends on the subtraction constants and on the slowly varying π-Y center-of-mass momenta. For each isotopic spin channel, we approximate z_I by a complex constant.[1,12] The calculations were done by adjusting the parameters R_I and z_I to fit the at-rest branching ratio[7]

$$\lambda^{-1} = \sigma_{\mathrm{abs}}(I = 1)/\sigma_{\mathrm{abs}}(I = 0) = 0.18 \pm 0.09,$$

the total cross section at 172 Mev/c ($k\sigma_{\mathrm{tot}}/4\pi = 0.7$ fermi), and the elastic and charge-exchange cross sections at 100 Mev/c and 172 Mev/c.

Two sets of solutions were found, corresponding to constructive or destructive Coulomb-nuclear interference in the angular distribution at 172 Mev/c (Table I). The elastic and charge-exchange cross sections resulting from the "constructive" solution are shown in Fig. 1. The energy dependence of $(k \cot\delta_I)^{-1}$ is shown in Fig. 2, where the (a+) solution of Dalitz and Tuan is also reported for comparison. Note that only the values of R_I and the ratio R_0/R_1 given by the "constructive" solution agree with those calculated theoretically. From this consideration we reject the "destructive" solution. Our principal result is that $k \cot\delta_I$ has a rather substantial energy dependence due to the two-pion exchange.

Table I. Two sets of parameters that fit the available experimental data. The solutions are characterized by the constructive or destructive nature of the Coulomb-nuclear interference in the angular distribution at 172 Mev/c.

Solution	Constructive	Destructive
R_1	$1.0 M_N^4$ fermi	$2.0 M_N^4$ fermi
R_0/R_1	-3.8	+1.50
z_0	$(1.34+i0.74) M_N^2$ fermi	$(0.93+i1.46) M_N^2$ fermi
z_1	$(0.67+i0.12) M_N^2$ fermi	$(-0.22+i1.00) M_N^2$ fermi
λ^{-1}	0.13	0.29

Figure 2 shows that $\mathrm{Re}(k \cot\delta_I)$ is positive for each isotopic spin state in the momentum range above 100 Mev/c. However, the real parts of the scattering lengths appear to have opposite signs, that of the $I=0$ state being small and negative.

Finally, we would like to remark that, by crossing symmetry, the two-pion contribution to the K^+-proton amplitude can be determined from the parameters R_0 and R_1. In this case there is a destructive interference and flattening of the cross section for a repulsive K^+-proton short-range interaction.

We are deeply indebted to Professor Geoffrey F. Chew and Professor Robert Karplus for many illuminating discussions.

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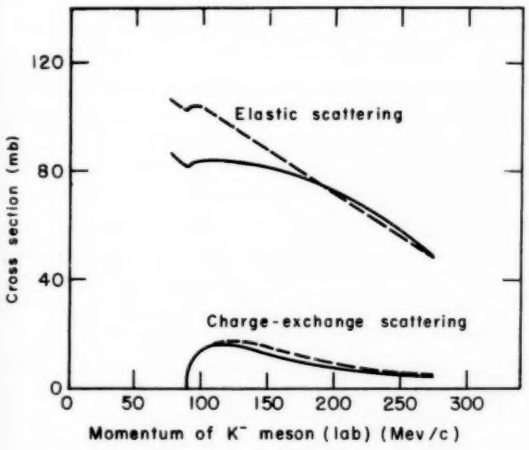

FIG. 1. Cross sections for elastic and charge-exchange K^--proton scattering. The (a+) solution of Dalitz and Tuan (dashed lines) is included for comparison.

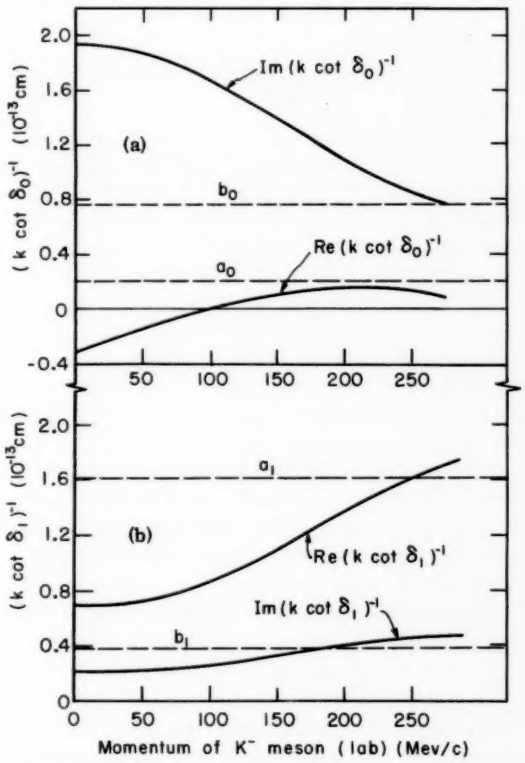

FIG. 2. Momentum dependence of the real and imaginary parts of $(k \cot\delta)^{-1}$, (a) for isotopic spin $I=0$, (b) for isotopic spin $I=1$. The dashed lines are the (a+) solution of Dalitz and Tuan.

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THEORY OF Σ-HYPERON DECAYS

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In this note we consider a model of the Σ hyperon as a bound state of a Λ^0 and a pion. By means of this model, we find it possible to explain some observed features of Σ decay simply in terms of the observed properties of Λ^0 decay.

For simplicity we shall consider the bound state to be an S state. This would lead to the following obvious features: (a) The spin of the Σ would be 1/2. (b) The isotopic spin of the Σ would be 1. (c) The relative parity of the Σ and Λ would be odd.[1]

Properties (a) and (b) are known to be satisfied.[2] Property (c) has not yet been investigated. However, a model of the Σ as a bound Λ and pion in a $P_{1/2}$ state will give similar predictions to those that follow.

We consider the following three decay modes of the Σ hyperons:

$$\Sigma^+ \to \pi^+ + n, \tag{1}$$

$$\Sigma^+ \to \pi^0 + p, \tag{2}$$

$$\Sigma^- \to \pi^- + n. \tag{3}$$

The mechanism for the decay is assumed to be the decay of the Λ hyperon within the Σ into a pion and a nucleon, with subsequent absorption by the nucleon, of either the pion within the Σ or the pion from the Λ decay, to form the appropriate final state of reactions (1), (2), or (3). The virtual decay of the Λ particle will be assumed to satisfy the $\Delta I = \frac{1}{2}$ rule and time-reversal invariance. This decay will be assumed to proceed with an amplitude A into an S-wave pion-nucleon system and with either an amplitude A or an amplitude $-A$ into a P-wave pion-nucleon system. For the present discussion, whose purpose is a qualitative picture of Σ decays, we will neglect multiple-scattering effects in the pion-nucleon systems of isotopic spin and total angular momentum $\frac{1}{2}$, and will consider the amplitude A as real. The approximate equality in magnitude of the S- and P-wave amplitudes is inferred from the assumption of a near maximal asymmetry parameter in the decay of a real Λ.[3] The positive and negative values of the P-wave amplitude correspond to the asymmetry parameter, α_Λ, equal to +1 and -1, respectively. The nucleon from the decay of the Λ will absorb a pion in a P wave relative to it. This may be the pion from P-wave decay of the Λ, in which case the absorption will be characterized, again approxi-

mately, by the real amplitude B. A second possibility is that the nucleon may absorb, via a P-wave interaction, the pion within the Σ, which is in an S state relative to the center of mass of the pion-nucleon system resulting from the Λ decay. This absorption will be characterized approximately by a real amplitude, B'.

With these definitions and making standard use of the Clebsch-Gordan coefficients, we construct the amplitudes for the reactions (1), (2), and (3).

$$\Sigma^+ = (\pi^+\Lambda)_S \to A\pi_S{}^+\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_S$$
$$\pm A\pi_S{}^+\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_P$$
$$\to \pm AB(\pi^+ n)_S, \qquad (4)$$

$$\Sigma^+ = (\pi^+\Lambda)_S \to A\pi_S{}^+\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_S$$
$$\pm A\pi_S{}^+\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_P$$
$$\to \tfrac{1}{3}\sqrt{2}\,AB'(\pi^0 p)_P \pm \tfrac{1}{3}\sqrt{2}\,AB'(\pi^0 p)_S, \qquad (5)$$

$$\Sigma^- = (\pi^-\Lambda)_S \to A\pi_S{}^-\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_S$$
$$\pm A\pi_S{}^-\{-(\tfrac{2}{3})^{1/2}(\pi^-p) + (\tfrac{1}{3})^{1/2}(\pi^0 n)\}_P$$
$$\to \pm AB(\pi^- n)_S$$
$$\pm \tfrac{2}{3}AB'(\pi^- n)_P \pm \tfrac{2}{3}AB'(\pi^- n)_S. \qquad (6)$$

In these equations the subscripts S or P on $(\pi\Lambda)$ and (πN) denote the orbital angular momentum of the pion and the baryon within the parenthesis; the subscripts S or P on the bracketed quantities, $\{\ldots\}$, denote the orbital angular momentum of the pion-nucleon systems within the brackets; the subscript S on the symbol π standing outside of the brackets denotes the orbital angular momentum of this pion relative to the center of mass of the pion-nucleon systems within the brackets. Products like AB are symbolic in that they will involve integrations over the momenta and a summation over the spin of a specific intermediate-state configuration.

The following predictions follow from our model, the aforementioned approximations, and Eqs. (4) - (6): (a) The decay $\Sigma^+ \to \pi^+ + n$ results in a pure S-wave final state and the asymmetry parameter for this mode is zero. (b) The asymmetry parameter in the decay $\Sigma^+ \to \pi^0 + p$ is nonzero and it has the same sign as the asymmetry parameter in Λ decay. Under the assumption that the asymmetry parameter in Λ decay is maximal, the decay $\Sigma^+ \to p + \pi^0$ proceeds with an equal admixture of S- and P-wave final states and consequently a maximal asymmetry parameter. (c) The decay $\Sigma^- \to \pi^- + n$, in general, involves a mixture of S- and P-wave final states. If we choose the single parameter in the theory, $AB'/AB = -3/2$, the decay $\Sigma^- \to \pi^- + n$ results in a pure P-wave final state and the asymmetry parameter for the mode is zero. It then follows that the three decay rates for the reactions (1), (2), and (3) are equal. If these three rates are not equal, the asymmetry parameter in the decay $\Sigma^- \to \pi^- + n$ will be nonzero.

We consider the leptonic decay modes (with nucleon emission) of the hyperons. It is immediately evident that the model contains the beta (and muon) decay of the Σ^- via the beta (and muon) decay of the Λ hyperon within the Σ^-:

$$\Sigma^- = (\pi^-\Lambda)_S \to a\pi_S{}^-(e^-\bar{\nu}p) \to\sim -(2/3)^{1/2}aB'(e^-\bar{\nu}n), \qquad (7)$$

where a is the amplitude for the virtual beta decay of the Λ. Similarly the beta (and muon) decay of the Σ^+ would in general be allowed if there existed a decay of the Λ^0 of the form $\Lambda^0 \to \pi^- + e^+ + n + \nu$. However, if one makes the simple additional assumption that all weak interactions proceed only through four-fermion couplings, then this mode of the Λ^0 is <u>not</u> allowed and the Σ^+ cannot undergo beta (or muon) decay. Also decays such as $\bar{K}^0 \to e^+ + \pi^- + \nu$ which have $\Delta S/\Delta Q = -1$ are forbidden. It is necessary to forbid such modes in the conventional theory, to avoid violating the $\Delta S \neq 2$ rule for the weak interactions.

We note that, as a crude exercise, we can calibrate the amplitude A from real Λ decay and amplitude B' from real Σ^- decay. Then the ratio of the rate for Λ beta decay to that for Σ^- beta decay may be obtained. Carrying out this exercise gives a value for this ratio of ~0.1, which is not very different from the ratio of ~0.17 obtained from phase space considerations alone.[4]

In conclusion, we state that the theory gives a simple basis for predicting a number of features of Σ decays that present experiments seem to indicate[5,6]: (a) Decay modes (1) and (3) seem to exhibit small asymmetry parameters and decay mode (2) a large asymmetry parameter. (b) As

a consequence of (a) and the $\Delta I = \frac{1}{2}$ rule, the decay rates for the reactions (1), (2), and (3) are approximately equal. (c) The Σ^- will undergo leptonic decay and the Σ^+ will not undergo leptonic decay (with nucleon emission) if one assumes that the weak interactions go only by way of four-fermion couplings. The theory predicts that the sign of the asymmetry parameter for the decay mode (2) is the same as that of the asymmetry parameter in Λ decay, and that the two asymmetry parameters are comparable in magnitude. For odd relative Σ - Λ parity, the present approximate treatment indicates that the decay mode (1) involves a pure S state and the decay mode (3) a pure P state; a measurement of the recoil neutron polarizations could check these consequences of the theory. The asymmetry parameter for the decay mode (3) may be nonzero if the three decay rates are not precisely equal.

If the Σ is a bound $P_{1/2}$ state of Λ^0 and a pion, then the only difference would be that the $\Sigma^+ \to \pi^+ + n$ decay would be all in P state rather than S state.

One of the authors (S. B.) wishes to thank Professors Sam Schweber and Kenneth Ford for their comments. The other would like to extend similar thanks to Professor G. Feinberg and Professor T. D. Lee.

[1]S. Barshay, Phys. Rev. Letters 1, 97 (1958).
[2]F. Eisler et al., Nuovo cimento 7, 222 (1958).
[3]1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 268.

[4]It is possible that the Σ^- beta decay will be suppressed relative to the Λ beta decay by the tendency of a four-fermion V-A interaction, which may mediate the virtual Λ beta decay within the Σ^-, to leave the proton with little recoil momentum, and hence largely in a S state relative to the pion within the Σ^-. This would diminish the absorption amplitude estimated crudely in eq. (7) by B'.

[5]P. Nordin et al., Phys. Rev. Letters 1, 380 (1958).
[6]R. L. Cool, B. Cork, J. W. Cronin, and W. A. Wenzel, Phys. Rev. 114, 912 (1959); P. Franzini et al., Bull. Am. Phys. Soc. 5, 224 (1960). The product of asymmetry parameter and average hyperon polarization is 0.02 ± 0.08 for decay mode (1), 0.62 ± 0.12 for decay mode (2), and 0.1 ± 0.17 for decay mode (3). The Σ^- and Σ^+ are produced in $\pi^- n$ and $\pi^+ p$ collisions, respectively, and are known to have a nonzero polarization owing to the substantial up-down asymmetry observed in the decay mode (2). The measurements on the Σ^+ are from Cool, Cork, Cronin, and Wenzel.

ELECTROMAGNETIC CORRECTIONS TO THE DECAYS OF THE MUON, O^{14}, AND THE NEUTRON*

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According to the conserved vector current hypothesis of Feynman and Gell-Mann[1] for the weak interactions, the vector coupling constant G_V should be the same for μ decay and nuclear β decay. However, the values of G_V determined from the experimental muon lifetime and the decay rate in the $0^+ - 0^+$ transition $O^{14}(\beta^+)N^{14*}$ differ by a few percent after inclusion of the radiative corrections discussed by a number of authors.[2] Kinoshita and Sirlin predict a muon lifetime of $\tau_\mu = (2.31 \pm 0.05) \times 10^{-6}$ sec on the basis of an O^{14} half-life of 72.5 ± 0.5 sec,[3] and a β^+ end-point kinetic energy of 1810 ± 8 kev.[4] With a recent revised end-point energy of 1800 ± 6.5 kev,[5] the predicted lifetime becomes $\tau_\mu = (2.26 \pm 0.05) \times 10^{-6}$ sec.[6] These numbers are to be compared with the observed lifetime $\tau_\mu = (2.21 \pm 0.005) \times 10^{-6}$ sec.[7] While the discrepancy may not be serious, it should be noted that the calculated radiative corrections are subject to certain ambiguities: the perturbation theory approach used in the calculations of reference 2 yields finite results in the muon case,[8] but the calculated corrections for nuclear β decay are ultraviolet divergent. The cutoff-dependent term in the correction to the decay rate, $\Delta\Gamma/\Gamma_0 = (3\alpha/2\pi) \ln(\Lambda^2/m_p m_e)$, was evaluated by choosing a cutoff Λ equal to the proton mass m_p, in the expectation that the proper inclusion of nuclear electromagnetic form factors would introduce a natural cutoff of the virtual photon contributions near this value. The resulting expression $\Delta\Gamma/\Gamma_0 = (3\alpha/2\pi) \ln(m_p/m_e) = 0.026$ represents the dominant part of the quoted radiative correction. This rather unsatisfactory situation makes it desirable to re-examine the problem from a somewhat different point of view. In the present paper, we wish to discuss the results of a calculation based on the techniques of

dispersion theory.

The S-matrix element for the process $\mu \to e + \nu + \bar{\nu}$ may be written

$$\langle e\nu\bar{\nu}|S|\mu\rangle = i(2\pi)^4\delta(\mu - e - \nu - \bar{\nu})(16e_0\mu_0\nu_0\bar{\nu}_0)^{-1/2}F(e\nu\bar{\nu};\mu), \quad (1)$$

where $\mu, e, \nu, \bar{\nu}$ denote the four-momenta of the particles. From invariance under proper Lorentz transformations, it follows that F has the general structure

$$F = (\bar{u}(\nu)\gamma_\alpha(1+\gamma_5)v(\bar{\nu}))\bar{u}(e)\{\gamma_\alpha[a(1+\gamma_5) + b(1-\gamma_5)] - i(\mu+e)_\alpha[c(1+\gamma_5)+d(1-\gamma_5)]\}u(\mu), \quad (2)$$

provided the usual two-component neutrino theory is assumed. For convenience, the free particle spinors are normalized according to

$$\bar{u}(p)\gamma_\alpha u(p) = 2ip_\alpha. \quad (3)$$

To lowest order in the weak interaction, the form factors, a, b, c, and d are functions of the single invariant parameter

$$s = -(\mu - e)^2 = -(\nu + \bar{\nu})^2. \quad (4)$$

In accordance with the conventional theory,[1] we require that, in the absence of electromagnetic interactions, $b = c = d = 0$, and a = constant. The assumption of microscopic causality leads, by the usual heuristic methods, to the conclusion that these functions are analytic in the complex s plane, except for singularities on the real axis. In the resulting dispersion relations for these functions, the absorptive parts may be determined from unitarity as a sum over all intermediate states in the scattering process, $\nu + \bar{\nu} \to \mu + \bar{e}$.

In contrast to the analogous problem in meson theory, all intermediate states of the form $\mu' + \bar{e}' + n\gamma$ produce singularities extending down to the threshold at $s = (m_\mu + m_e)^2$, and none can be neglected on the basis of its remoteness from the region of physical interest. However, the fine structure constant α appears to higher than first power in the contribution from all states save those of the form $\mu' + \bar{e}'$ [Fig. 1(a)]. When only these states are retained in the absorptive part, the dispersion relations become a set of integral equations for the invariant functions, accurate to order α. The amplitude for μ-$\bar{e}$ scattering appearing in these equations is

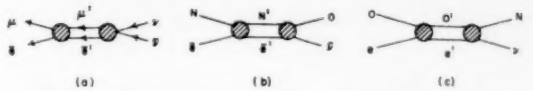

FIG. 1. The lowest order dispersion diagrams encountered in the calculation of the absorptive part of the decay amplitude for (a) the muon, and (b), (c) the $0^+ \to 0^+$ decay $O^{14} \to N^{14*}$.

therefore evaluated in Born approximation, and the equations solved by a single iteration, taking for zeroth order the values: a = const, $b = c = d = 0$. If unsubtracted dispersion relations are assumed for b, c, and d, convergent integrals are obtained for these functions. However, the imaginary part of a contains a term

$$(s - m_e^2 - m_\mu^2)(4p^2s)^{-1/2}\int_0^{4p^2}\frac{dq^2}{q^2+\lambda^2}, \quad (5)$$

where $4p^2s = [s - (m_\mu + m_e)^2][s - (m_\mu - m_e)^2]$, and λ is the usual fictitious photon mass introduced to circumvent the infrared problem. For large s, this term behaves as $\ln(s/\lambda^2)$. Consequently, a dispersion integral with a single denominator would not converge, and it is necessary to use a dispersion relation for $a(s)$ with one subtraction,

$$a(s) = a(s_0,\lambda) + \frac{s - s_0}{\pi}\int_{(m_\mu+m_e)^2}^{\infty}\frac{ds'\,\mathrm{Im}a(s')}{(s'-s)(s'-s_0)}, \quad (6)$$

in which there appears a phenomenological constant $a(s_0, \lambda)$.

The dependence of the decay rate on terms which diverge in the limit $\lambda \to 0$ must be removed when contributions from the bremsstrahlung process $\mu \to e + \nu + \bar{\nu} + \gamma$ are introduced.[9] With an arbitrary choice of subtraction point s_0, this would require that the constant $a(s_0, \lambda)$ itself contain an infrared divergence. However, explicit cancellation is found to occur if and only if the choice $s_0 = (m_\mu - m_e)^2$ is made. This circumstance makes it highly attractive to define the renormalized coupling constant for μ decay as[10]

$$G_\mu = \sqrt{2}\,a((m_\mu - m_e)^2). \quad (7)$$

Using this definition, we obtain for the electron spectrum results (DLM) differing from those of

perturbation theory (KS)[2] only by the term

$$\left(\frac{dN}{dx}\right)_{\mathrm{DLM}} - \left(\frac{dN}{dx}\right)_{\mathrm{KS}} = \frac{5\alpha}{4\pi}\left(\frac{11}{5} - \ln\frac{m_\mu}{m_e}\right)\left(\frac{dN}{dx}\right)_0, \tag{8}$$

where $(dN/dx)_0$ denotes the statistical spectrum. The resulting electron decay rate is decreased by 1.32% from its value in the absence of electromagnetic corrections. However, to order α, the Michel parameter is unchanged from the result of the perturbation theory calculations,[2] $\rho_{\mathrm{eff}} \approx 0.708$.

Similar techniques may be applied to the decay of the neutron and to the $0^+ \to 0^+$ transitions $O^{14}(\beta^+)N^{14*}$, $Al^{26*}(\beta^+)Mg^{26}$, and $Cl^{34}(\beta^+)S^{34}$. For $0^+ \to 0^+$ transitions, the S-matrix element in covariant form has the structure

$$\langle \bar{e}\nu p_2 | S | p_1 \rangle = i(2\pi)^4 \delta(p_1 - p_2 - \bar{e} - \nu)(16\bar{e}_0\nu_0 p_{10} p_{20})^{-1/2} \times F(\bar{e}\nu p_2; p_1), \tag{9}$$

where $p_{1,2}$ denote the nuclear 4-momenta, and

$$F = i[A(s)(p_1 + p_2)_\alpha + B(s)(p_1 - p_2)_\alpha] \times (\bar{u}(\nu)\gamma_\alpha v(\bar{e})), \tag{10}$$

In addition to the indicated dependence on the invariant parameter $s = -(p_2 + \bar{e})^2$, the functions A and B depend on the momentum transfer $Q^2 = (p_1 - p_2)^2$. This dependence may be adequately accounted for, however, by the usual multipole expansion of the nuclear matrix elements.[11] We therefore utilize dispersion relations for A and B as functions of s, holding Q^2 fixed at the static nuclear value, $-(M_1 - M_2)^2 = -(\Delta E)^2$. In the absence of electromagnetic interactions, we require $B(s) = 0$ and $A(s) = \mathrm{const} \times M$, where M is the usual nuclear matrix element for the allowed Fermi transition. The contributions which we included in the iterative procedure are represented, for the case of O^{14}, by the dispersion graphs shown in Figs. 1(b) and 1(c). In the amplitude for scattering of the β ray by the nucleus, nuclear electromagnetic form factors may be introduced. The dominant term in the absorptive part appears as a contribution to $\mathrm{Im}A(s)$, and is again of the type given by (5), with appropriate change of the masses, and with an electromagnetic form factor $F(q^2)$ appearing under the integral sign. For large s, this quantity now approaches a constant value, instead of diverging logarithmically, but a single subtraction is still necessary in the dispersion relation for $A(s)$.[12] Explicit cancellation of the infrared-divergent parts now requires that the subtraction point s_0 be chosen to correspond (in the nucleon rest frame, and in the static limit) to the (unphysical) positron energy

$$\bar{e}_0 \approx 2m_e[\pi(2Z+1)]^{-1}, \tag{11}$$

where Z is the atomic number of the final nucleus. In analogy with the muon case, we treat the value of A at this point as a phenomenological constant, and define the renormalized coupling constant G_V for the $0^+ \to 0^+$ transitions by

$$A(s_0) = (G_V/\sqrt{2})M. \tag{12}$$

The radiative corrections[13] to the spectrum of O^{14} calculated by this means are shown in Fig. 2. In addition to electromagnetic (radiative) effects, we have considered various other corrections to the decay rates which might be expected to be of the same order of magnitude. These corrections are summarized in Table I. Final numerical results for the effective ft values for the transitions considered are also presented in Table I. Apart from numerical factors, the constant G_V is in each case defined to be the value of the subtraction constant appearing in the dispersion relations, with the choice of subtraction point dictated by the requirement of cancellation of

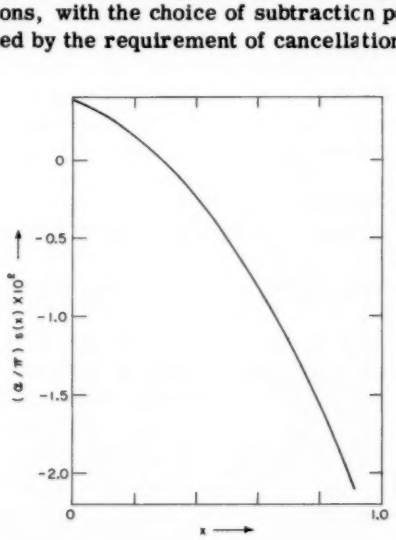

FIG. 2. The radiative electromagnetic correction (see reference 13) to the positron spectrum in the decay of O^{14}. The radiative contribution to the spectrum is $(dN/dx)_{\mathrm{rad}} = (\alpha/\pi)s(x)(dN_0/dx)$, where dN_0/dx is the statistical spectrum, not including the Fermi factor.

Table I. Corrections and ft values for nuclear beta decays.

	n	O^{14}	Al^{26*}	Cl^{34}
Corrections(%)				
Electromagnetic (radiative)[a]	-0.56	-1.03	-1.25	-1.62
Nuclear electromagnetic form factors	-0.01	+0.106	+0.328	+0.639
Competition from K capture	···	0.090	0.078	0.068
Electron screening	···	0.093	0.113	0.127
"Second forbidden" nuclear matrix elements[b]	···	-0.024	-0.068	-0.124
Coulomb corrections to the nuclear matrix element[c]	···	-0.35	+0.40 ± 0.2	1.0 ± 0.2
Total corrections[d]	-0.57	-1.12	-0.40 ± 0.2	+0.09 ± 0.2
End-point energy (total), kev[e]	1293 ± 1	2311 ± 6.5	3713 ± 10	5011 ± 30
$f(Z, E_m)$[f]	1.688	41.82	467.7	2036
$f_{eff}(Z, E_m)$, including all corrections	1.678	41.34	465.8	2038
Half-life, sec[e]	702 ± 18	72.5 ± 0.5	6.60 ± 0.06	1.53 ± 0.02
$f_{eff}(Z, E_m)t$, sec	1178	2998 ± 50	3074 ± 53	3118 ± 110
G_V, 10^{-49} erg cm^3	1.35 ± 0.04[g]	1.43 ± 0.01	1.41 ± 0.01	1.40 ± 0.01

[a]See reference 13.

[b]See reference 9.

[c]Taken from W. M. MacDonald, Phys. Rev. 110, 1420 (1958), for O^{14} and Cl^{34}, and estimated for Al^{26*} on the basis of those values. The calculation for Cl^{34} was incomplete, but the omitted terms are expected to decrease slightly the magnitude of the 1.2% correction quoted. We therefore assume a correction of (1.0 ± 0.2) %.

[d]The greatest theoretical uncertainties are probably in the Coulomb corrections to the nuclear matrix elements for Al^{26*} and Cl^{34} (see preceding remarks).

[e]The end-point energies and half-lives are those summarized by O. C. Kistner and B. M. Rustad, Phys. Rev. 114, 1329 (1959), with the exception of the O^{14} end point (see references 5 and 6).

[f]Calculated by numerical integration using Tables for the Analysis of Beta Spectra, National Bureau of Standards Applied Mathematics Series, No. 13 (U. S. Government Printing Office, 1952).

[g]Calculated using $(G_A/G_V)^2 = 1.56 \pm 0.14$, as determined from the value of the neutron spin-electron momentum correlation coefficient, $A = -0.11 \pm 0.02$ [M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. Letters 1, 324 (1958)].

the infrared-divergent parts, as previously indicated.

In interpreting these results, one should note that the present approach necessitates the abandonment of the usual formulation of universality, based on the equality of bare coupling constants in a Lagrangian theory. In this sense, there is no compelling reason for believing in the equality of the renormalized constants G_V and G_μ as we have defined them. On the other hand, the necessity of avoiding infrared divergences evidently permits only one natural definition of the renormalized coupling constant, that which we have used. It is therefore of interest to inquire into the consequences of assuming that $G_V = G_\mu$. Using the value of G_V derived from the O^{14} data,[5,6] this assumption leads in fact to a value for the muon lifetime,

$$\tau_\mu = (2.23 \pm 0.05) \times 10^{-6} \text{ sec}, \qquad (13)$$

in excellent agreement with experiment.[7] Further discussion of these results, and details of the techniques employed in the calculations, will be presented in a paper now being prepared.

We wish to acknowledge numerous valuable conversations with R. Blankenbecler, R. Behrends, A. Sirlin, and G. C. Wick. We also wish to thank I. Cole for computational aid.

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[1]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); M. Gell-Mann, Phys. Rev. 111, 362 (1958).

[2]T. Kinoshita and A. Sirlin, Phys. Rev. 113, 1652 (1959); S. M. Berman, Phys. Rev. 112, 267 (1958); R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. 101, 866 (1956).

[3]J. B. Gerhart, Phys. Rev. 95, 288 (1954), $t = 72.1 \pm 0.4$ sec. R. Sherr, J. B. Gerhart, H. Horie, and W. F. Hornyak, Phys. Rev. 100, 945 (1955), (0.60 ± 0.10)% branching ratio to the ground state of N^{14}.

[4]D. A. Bromley, E. Almqvist, H. E. Gove, A. E. Litherland, E. B. Paul, and A. J. Ferguson, Phys. Rev. 105, 957 (1957).

[5]R. O. Bondelid, J. W. Butler, C. A. Kennedy, and A. del Callar (to be published) (private communication from R. O. Bondelid).

[6]It is of considerable importance that the discrepancy between the $C^{12}(He^3, n)O^{14}$ threshold measurements of references 4 and 5 be resolved. In the present paper, we will use the $O^{14}(\beta^+)N^{14*}$ end-point kinetic energy of 1800 ± 6.5 kev calculated from the threshold value of reference 5.

[7]J. Fischer, B. Leontic, A. Lundby, R. Meunier, and J. P. Stroot, Phys. Rev. Letters 3, 349 (1959), $\tau_\mu = 2.20 \pm 0.015\ \mu$sec. R. A. Reiter, T. A. Romanski, and R. B. Sutton, post-deadline paper, Washington APS Meeting, 1960, $\tau_\mu = 2.211 \pm 0.003\ \mu$sec. V. L. Telegdi, R. A. Swanson, R. A. Lundby, and D. D. Yovanovitch (private communication from Dr. Yovanovitch), $\tau_\mu = 2.208 \pm 0.004\ \mu$sec.

[8]We are indebted to Professor R. H. Dalitz for pointing out to us that the results of the perturbation theory calculations for the muon are ambiguous to the extent that one is at liberty to regularize in different ways the muon and electron self-energy graphs and the vertex graphs, thereby adding finite constant terms to the decay amplitude. The choice of reference 2 corresponds to regularizing only the photon propagators.

[9]To lowest order in α, a consistent treatment of the decay accompanied by inner bremsstrahlung yields results identical with those of perturbation theory, with the usual weak-coupling constant G replaced by $\sqrt{2}\,a(s_0, \lambda)$.

[10]The factor $\sqrt{2}$ is introduced to conform to the notation for G of reference 1.

[11]M. Morita, Phys. Rev. 113, 1584 (1959).

[12]The failure to obtain a finite result for $A(s)$ when the electromagnetic form factor is included in the calculation of $\mathrm{Im}A(s)$ contradicts previous conjectures (see the first two papers of reference 2). We remark, however, that the form factor is necessary and sufficient to secure convergence in the integral for $B(s)$ and in the contributions to the neutron decay amplitude involving interactions of the electron with the anomalous magnetic moments of the nucleons. These form-factor-dependent terms do not contribute significantly to the corresponding decay rates.

[13]A term in the total electromagnetic correction of the form $-Z\alpha\pi/v_e$ is identified in the usual manner with the term of order $Z\alpha$ in the expansion of the Fermi factor for the positron decay, and is omitted from the corrections designated as radiative.

SOME APPLICATIONS OF THE GENERALIZED UNITARITY RELATION

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Recent studies of the analytic continuations of perturbation theory amplitudes have led to a greatly increased understanding of their properties.[1-10] Landau[4] showed the existence of a class of singularities which are a generalization of the ordinary thresholds for the onset of new physical processes. He derived two conditions for determining the location of these singularities: (1) Each of the intermediate particles (which correspond to lines joining otherwise disjunct subgraphs of the original Feynman graph) is on the mass shell. (2) The four-momenta of these intermediate particles satisfy a geometrical relation which generalizes the requirement, for the ordinary thresholds, that the intermediate particles all be at rest in the center-of-mass frame.

Applications of dispersion relations have all made use of the fact that the discontinuity across a branch cut starting from an ordinary physical threshold is obtained from the unitarity condition. It has been shown, for all finite order graphs, that a generalized unitarity relation gives the discontinuity across a branch cut which starts from any Landau singularity.[11] In this note we point out, by means of some simple examples, that this generalized unitarity relation greatly augments the power of analytic continuation methods.

The rule for calculating the discontinuity of a graph is that in the integral over the virtual four-momenta, for each line of the "reduced graph" which determines the singularity the factor $(M^2 - q^2)^{-1}$ is replaced by $2\pi i\delta(q^2 - M^2)$. In other words, the subgraphs which comprise the vertices of the reduced graph are considered only for the case that the lines leading into them represent free particles. The products of the factors $\delta(q^2 - M^2)$ and d^4k for the reduced graph are analogous to phase space volume elements.

Certain reduced graphs lead to poles rather than branch points; in these cases the residue is obtained by considering the lines to be on the mass shell. The poles discussed by Chew and Low[12] are simple examples.

The Landau singularities are, in fact, all the singularities of a finite order graph, apart from trivial exceptional branch points that may occur on secondary Riemann sheets.[11] Since the discontinuities and residues of a given graph are expressible in terms of lower order graphs, the generalized unitarity relation can be used to generate the perturbation series expansion, when it is supplemented by appropriate assumptions about the number of necessary subtractions.[13] Therefore, the generalized unitarity relation may be considered to embody the entire content of a convergent perturbation expansion.

As a first illustration, we consider the anomalous threshold of the reaction $\pi+\pi\to\Sigma+\Sigma$, which has already been discussed by Mandelstam.[14] All graphs which possess the anomalous threshold in question have the structure shown in Fig. 1(a). The circles stand for any graphs which contribute to the $(\pi\Sigma\Lambda)$ vertex, or to π-π scattering. When we sum over all graphs which contribute to the discontinuity above the anomalous threshold, these become the renormalized coupling constant and the elastic scattering amplitude. We follow Mandelstam and treat the particles as spinless and chargeless; the discontinuity of $T_{\pi\Sigma}(E,\cos\theta)$ is then

$$[T_{\pi\Sigma}(E,\cos\theta)] = (2\pi^{-1})g^2\int d^4k\, T_{\pi\pi}(E,\cos\theta') \times \delta(q_+^2 - M_\pi^2)\delta(q_-^2 - M_\pi^2)\delta(q_0^2 - M_\Lambda^2), \quad (1)$$

where $q_\pm = \frac{1}{2}P \pm k$ and $q_0 = p_\Sigma - k$ (we denote by P the total four-momentum, and by $2p_\Sigma$ the difference of the four-momenta of the Σ's). If we expand in partial waves, we obtain

$$[T_{\pi\Sigma}{}^{l}(E)] = g^2(8p_\Sigma E)^{-1}P_l(x)\,T_{\pi\pi}{}^{l}(E). \quad (2)$$

Here x, the cosine of the angle between the Σ and the intermediate π, is given by $M_\Lambda{}^2 + p_\pi{}^2 + p_\Sigma{}^2 = 2p_\pi p_\Sigma x$. At the anomalous threshold, $x=1$; as E increases to $2M_\pi$, $x\to+\infty$. Equation (2) for $l=0$ corresponds to Mandelstam's result, but it is here shown to include the contribution of all orders of perturbation theory.

The electromagnetic form factor of the deuteron has a first threshold corresponding to the reduced diagram 1(b).[1] When all graphs are summed, the circles correspond to the asymptotic Bethe-Salpeter amplitude and to the nucleon form factor. Integration leads to the usual zero-range potential result, with some relativistic corrections. The next threshold corresponds to graph 1(c), which has an intermediate pion. The pion production vertex has two nucleon pole terms, which appear to be dominant. It follows from a theorem proved in reference 11 that in calculating the discontinuity associated with the reduced graph 1(c), these poles never lie in the region of integration, but they are actually quite close to the edge. The pole terms can be interpreted as the lowest order correction to the wave function arising from the finite range of the force. The remainder of the pion production vertex gives a "nonadditivity" correction.

The technique described above can also be used to calculate, and to give a physical interpretation to, the Mandelstam spectral functions. These correspond to a division of a graph into four (or more) disjunct parts, with each external line being attached to a separate part. Consider first the reduced nucleon-nucleon scattering graph 2(a). The corresponding Mandelstam spectral function is just the renormalized fourth order perturbation theory result.[2,3]

Another reduced two-meson exchange graph is shown in Fig. 2(b). When all contributing Feyn-

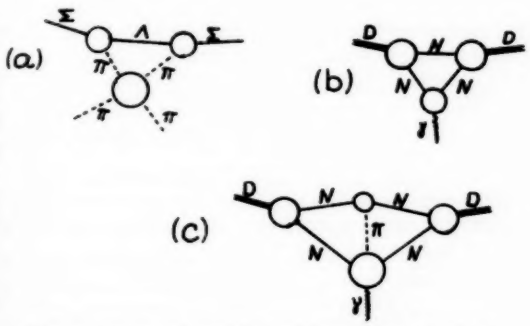

FIG. 1. Reduced graphs corresponding to some "anomalous" thresholds.

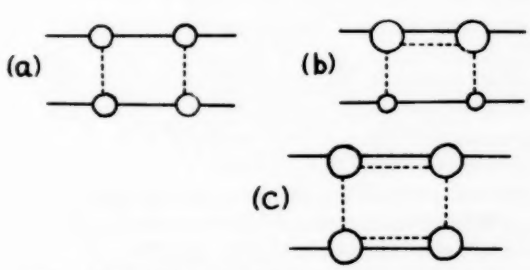

FIG. 2. Reduced graphs corresponding to some Mandelstam spectral thresholds for nucleon-nucleon scattering. The solid lines represent nucleons, and the dashed lines represent pions.

man graphs are summed, one of the circles becomes the meson-nucleon scattering amplitude; by paying careful attention to the branch onto which the amplitude is continued, it can be seen that the other upper circle is the conjugate amplitude. Hence we find, for 2(b),

$$\rho(s,t) = (2\pi i)^{-3}\int d^4k_1\, d^4k_2\, g^2(\gamma\cdot P - M)\delta_1\ldots\delta_5 \times T_{\pi N}^{\dagger}(s',t')T_{\pi N}(s',t''), \quad (3)$$

where $\delta_1\ldots\delta_5$ are the delta functions associated with the five lines of the reduced graph, P is the momentum of the intermediate nucleon in the lower part of the graph, and $s' = (q_4+q_5)^2$ is the energy variable of the meson-nucleon scattering. Let us introduce a dummy integration of the form $\int\delta(s' - (q_4+q_5)^2)ds'$. For s' below the meson production threshold, we find by analytically continuing the ordinary unitarity relation, that

$$(2\pi i)^{-2}\int d^4k_2\,\delta_4\delta_5 T_{\pi N}^{\dagger}(s',t')T_{\pi N}(s',t'') = 2i\ \mathrm{Im}T_{\pi N}(s',t). \quad (4)$$

Hence

$$\rho(s,t) = \pi^{-1}\int ds'\int d^4k_1\,\delta_1\ldots\delta_3\delta(s' - (q_4+q_5)^2) \times g^2(\gamma\cdot P - M)\ \mathrm{Im}T_{\pi N}(s',t). \quad (5)$$

The integration over d^4k_1 is now identical to that which occurs in fourth order perturbation theory,[11] and leads to

$$\rho(s,t) = g^2\int_{s_0}^{s_1} ds'(\gamma\cdot P - M)\,\mathrm{Im}T(s',t)/V(s,t,s'). \quad (6)$$

The denominator V is proportional to the volume of a certain four-dimensional figure. The s' integration goes between the limits $s_0 = (M_N+M_\pi)^2$ and a maximum s_1 determined by the equation $V(s,t,s_1) = 0$.

If we consider also the reduced graphs with two or more mesons (or other particles) passing between the upper circles in graph 2(b), we find, by using the unitarity condition for the inelastic scattering situation, that these graphs give such a contribution that Eq. (6) is also correct for $s' > (M_N+2M_\pi)^2$. Graph 2(c) may be analyzed by the same method and leads to a result similar to (6), except that the imaginary part of a meson scattering matrix is associated with each nucleon. The crossed reduced graphs may be obtained from crossing symmetry.

We see that the contribution of all two-meson exchange graphs to the Mandelstam spectral functions can be written down in terms of the meson-nucleon scattering amplitude. It is clear, in particular, that meson-meson interactions do not contribute to nucleon-nucleon scattering and to meson-nucleon scattering in fundamentally different ways. Of course, it is unlikely that $\mathrm{Im}T_{\pi N}(s',t)$ in the spectral region can be obtained from the experimental values by a model-independent extrapolation.

It should be noted that there are some Feynman graphs, involving the $\lambda\phi^4$ term in the Lagrangian, which do not contribute to the spectral functions. If a subtracted form of the Mandelstam representation is used, these terms are absorbed.

The cases (such as Σ - Σ scattering) in which the Mandelstam representation is invalid are not greatly different from nucleon-nucleon scattering; there are spectral functions determinable in terms of simpler processes, but they must be integrated over complex surfaces.

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

BOLTZMANN-VLASOV EQUATION FOR A QUANTUM PLASMA. Oldwig von Roos, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California (Received April 11, 1960).

In this paper a new equation is derived which is a quantum mechanical analog of the classical collisionless Boltzmann equation. To these ends a new quantum mechanical distribution function is employed which is particularly useful in this case. Quantum corrections for longitudinal plasma oscillations are evaluated for a low-density plasma, and it is found that the leading contribution ($\sim\hbar^2$) is due to exchange (Pauli principle).

STOCHASTIC EQUATIONS FOR NONEQUILIBRIUM PROCESSES. P. M. Mathews,* I. I. Shapiro, and D. L. Falkoff,† Department of Physics, Brandeis University, Waltham, Massachusetts (Received August 3, 1959).

A system of weakly interacting particles is described by a time-dependent joint probability distribution in the occupation numbers of the individual particle states. The "master" equation for the distribution is obtained by considering the time evolution of the system as a Markoff process with transition probabilities per unit time given by first-order quantum mechanical perturbation theory. This is done for particles obeying classical and quantum statistics. The resulting equations include the usual rate equations for the average occupation numbers as special cases; but they also yield all higher moments and correlations in the occupation numbers. The general solution and its properties are discussed for the case in which a relaxing subsystem interacts via binary collisions with a larger system having a fixed but not necessarily thermal distribution. The explicit solution for the joint distribution in occupation numbers for all time is constructed for the case of identical harmonic oscillators which have an arbitrary initial distribution. These interact via binary collisions with a reservoir of similar oscillators, the coupling being linear in each oscillator coordinate. This model is also generalized and solved for a case in which the number of interacting particles is not conserved.

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STUDY OF SUPERFLUIDITY IN LIQUID He BY ION MOTION. F. Reif and Lothar Meyer, Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received April 15, 1960).

Ions produced in liquid helium by ionization with α particles have been exploited as microscopic probe particles to study the properties of the superfluid. A time-of-flight method was used to measure directly the drift velocity u of the ions in the liquid for various values of the applied electric field $\mathcal{E}$ and at temperatures T down to 0.5°K. The field-independent mobility $\mu^{(0)} \equiv u/\mathcal{E}$ obtained in the limit of sufficiently small fields increases very rapidly below the λ point. Its temperature dependence over most of the temperature range below 2°K is of the form $\mu^{(0)} = \alpha \exp(\Delta'/kT)$ and can be explained by the scattering of the ions from the collective excitations (rotons) present in the fluid. The nonlinear dependence of u on $\mathcal{E}$ at higher field strengths was also investigated and suggests the possibility of ions creating excitations in the quantum field as an important inelastic scattering process at sufficiently large fields. Some additional experiments are suggested.

NEUTRON SCATTERING BY FLUIDS AND THE LAW OF CORRESPONDING STATES. George H. Vineyard, Brookhaven National Laboratory, Upton, New York (Received April 15, 1960).

It is shown that the classical law of corresponding states implies correspondences in the time-displaced pair correlations of fluids, and also in the cross sections for neutron and x-ray scattering. A measurement of the scattering at one temperature and pressure by one fluid of a class of fluids can thus be used to determine the

scattering at corresponding temperatures and pressures of all fluids of the class. Other applications, in the study of molecular fluids and in the separation of the self-correlation from the general correlation, are pointed out.

EXPERIMENTAL PLASMA FLOW INTO A VACUUM MAGNETIC CUSP FIELD. F. R. Scott and R. F. Wenzel, John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic, San Diego, California (Received April 11, 1960).

A directed dense plasma has been observed to penetrate and be detained by a vacuum cusp magnetic field. A gross measurement of the directed velocity of the deuterium plasma was $(8.5 \pm 0.5) \times 10^6$ cm/sec. Ion density in the central region of the cusp was measured to be 8.5×10^{16} at 16 μsec after injection and had a decay constant of 13.5 μsec. Magnetic probe measurements are shown which indicate the gross reaction of the magnetic field in the injected plasma. Observation of impurity emission indicates that the plasma electrons cool rapidly and that the collision-dominated plasma is in contact with the container walls at the cusp ring.

FERMI SURFACE AND SOME SIMPLE EQUILIBRIUM PROPERTIES OF A SYSTEM OF INTERACTING FERMIONS. J. M. Luttinger, University of Pennsylvania, Philadelphia, Pennsylvania (Received March 28, 1960).

It is shown that certain analytical properties of the propagators of many-fermion systems lead rigorously to the existence of sharp discontinuities of the momentum distribution at absolute zero. This discontinuity in the momentum distribution is used to define a Fermi surface for a system of interacting fermions. It is shown that the volume of this surface in momentum space is unaffected by the interaction. The same analytic properties are shown to lead by direct statistical mechanical arguments to simple expressions for the low-temperature heat capacity, the spin-paramagnetism, and the compressibility of the system. These expressions are very analogous to the corresponding expressions for noninteracting particles. Finally, it is shown how the whole formalism may be generalized when an external periodic potential is present (band case).

EQUATION OF STATE OF A BOSE-EINSTEIN SYSTEM OF PARTICLES WITH ATTRACTIVE INTERACTIONS. Kerson Huang, Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received March 18, 1960).

The equation of state in the grand canonical ensemble is calculated for a system of Bose-Einstein particles with hard-sphere repulsive interactions and weak long-range attractions. The energy levels used in this calculation are modified forms of those derived in an earlier paper. The calculation is carried out in the limit of no interactions, and attention is focused on the thermodynamic phases of the system. It is shown that the gross features of the equation of state of He^4 are reproduced. There are the phases: gas, liquid I, and liquid II. The phase transitions between gas and the two liquids are first order transitions. The transition region for the gas-liquid I transition terminates in a critical point. The transition between liquid I and liquid II is the Bose-Einstein condensation. Liquid II has a negative coefficient of thermal expansion. Across the transition between liquids I and II the specific heat is discontinuous in value. In the limit of no interactions, the critical point recedes towards zero temperature, zero pressure, and infinite volume.

CORRECTION TO THE DEBYE-HÜCKEL THEORY. D. L. Bowers* and E. E. Salpeter, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received April 8, 1960).

The problem of a gas of particles all of the same charge, imbedded in a neutralizing medium of uniformly distributed charge of the opposite sign, is considered in terms of classical statistical mechanics. If a dimensionless parameter ϵ, roughly the inverse of the number of particles contained inside a Debye sphere, is small compared to unity the Debye-Hückel theory is a good first approximation. For this case the corrections in the next order in ϵ are derived for the potential of mean force and the correlation energy. It is shown how this correction has to be modified for very small particle separation; the expansion in powers of ϵ is not strictly a Taylor expansion and factors such as $\ln\epsilon$ appear in the higher terms. Methods are given for

numerical calculation of some auxiliary functions even when the parameter ϵ is not small.

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MICROWAVE BREAKDOWN OF AIR IN NONUNIFORM ELECTRIC FIELDS. Philip M. Platzman and Elsa Huber Solt, Hughes Research Laboratories, Malibu, California (Received April 11, 1960).

A new approach to the problem of diffusion-controlled breakdown in nonuniform fields is presented. Two specific cases are considered in detail. The geometrical configurations involved are, respectively, a rectangular microwave cavity with a small hemispherical boss on one of its walls and without a boss. The theoretical predictions of the method are compared with a series of microwave breakdown measurements in air. The theoretical results are found to be in good agreement with the observed experimental data.

DIFFUSION RATE OF Li IN Si AT LOW TEMPERATURES. E. M. Pell, General Electric Research Laboratory, Schenectady, New York (Received April 4, 1960).

The method of ion drift in the electric field of an n-p junction has been used to measure the diffusion constant of Li in Si between 25°C and 125°C, taking particular care to avoid chemical and electrical interactions which might affect the results. When these data are combined with previous high-temperature data, there is obtained $D = (2.5 \pm 0.2) \times 10^{-3} \exp[-(0.655 \pm 0.01) e/kT]$ cm^2/sec, the data extending over eight decades in D. The results are compared with those from ion-pair relaxation experiments, and it is shown that the latter are consistent with the ion drift results.

VIBRATIONAL MODES NEAR IMPURITIES. Herbert B. Rosenstock and Clifford C. Klick, U. S. Naval Research Laboratory, Washington, D. C. (Received March 25, 1960).

In many cases, the interaction between the electronic state of an impurity in a solid and the motion of the atoms in the lattice is controlled by the relative motion of the impurity with respect to its nearest neighbors. We therefore examine the nature of the vibrations of the lattice near an impurity. In particular, two problems are considered: first, the direct absorption of light by the "localized" mode introduced by the presence of the impurity, and the relation of its frequency to that of the "reststrahl" absorption; second, the relative importance of the "localized" modes and of the "lattice" modes in producing the observed broadening of the spectra due to electronic transitions of the impurity center. It is found that the large number of lattice modes together produce a mean displacement of the same order of magnitude as do the very few local ones. The implications for the "configuration coordinate" model for phosphors are examined; both theory and experiment suggest that if only one mode, or group of modes, are effective in producing broadening, their frequency is substantially lower than that of the localized modes which are optically active.

THEORY OF THE DIELECTRIC PROPERTIES OF THE ALKALI HALIDES. E. E. Havinga, Philips Research Laboratories, N. V. Philips' Gloeilampenfabrieken, Eindhoven, Netherlands (Received April 11, 1960).

In this paper some relations between dielectric properties of diagonal cubic ionic crystals are derived on the basis of the shell model for ions of Dick and Overhauser. The relations, which contain no model constants, are in a good agreement with experimental data. Also it is shown that Dick and Overhauser overestimated the number of electrons in the shells of the ions, which accounts for the failure of their quantitative treatment. In the Appendix the paper of Hanlon and Lawson on the same subject is discussed.

ELECTRON SCATTERING IN A HIGH MAGNETIC FIELD. Arnold H. Kahn, National Bureau of Standards, Washington, D. C. (Received February 23, 1960; revised manuscript received May 6, 1960).

Electrical conductivity in a strong magnetic field is calculated for the case of scattering by delta-function impurities. The impurity concentration is taken as sufficiently weak that collision broadening may be neglected. The scattering by an individual center is solved exactly

rather than by perturbation theory. As a result, transition probabilities for an electron at the bottom of a Landau level vanish, rather than diverge. Expressions are given for the longitudinal and transverse conductivities in the oscillatory range, and in the quantum limit range for degenerate and nondegenerate statistics. The relation of this theory to those employing collision broadening is discussed.

CHARGE CARRIER PRODUCTION AND MOBILITY IN ANTHRACENE CRYSTALS. R. G. Kepler, Central Research Department, Experimental Station, E. I. du Pont de Nemours and Company, Wilmington, Delaware (Received April 15, 1960).

The drift mobilities of electrons and holes in anthracene crystals have been measured using a pulsed photoconductivity technique. The mobilities found at room temperature vary from about 0.3 cm^2/volt sec to about 3 cm^2/volt sec, depending on the crystal orientation, and the mobilities increase as the temperature is lowered. The wavelength dependence of the number of charge carriers produced by a pulse of light, as well as other experimental data, indicates that the charge carriers are not produced in the interior of anthracene crystals, but that they are released from a surface layer of the crystal either directly by photons or by excitons which migrate to the surface.

CROSS RELAXATION IN RUBY. W. B. Mims and J. D. McGee, Bell Telephone Laboratories, Murray Hill, New Jersey (Received April 6, 1960).

A pulsed microwave method has been used to study paramagnetic relaxation in synthetic ruby at Cr/Al concentrations from 0.02% to 0.3%, and over a wide range of fields and angles with respect to the crystal axis. The experimental frequency was 7.17 kMc/sec. At settings for which one interval between energy levels was twice as large as another, decay traces with two characteristic periods were observed. The more rapid decay was independent of temperature, and is attributed to a cross relaxation process involving three spins. Similar behavior became apparent at all concentrations whenever two intervals approach the same value. At 0.3%, two-period decay traces were observed for any arbitrary field and angle setting, indicating, at this concentration, a general cross relaxation between the Zeeman levels in times of 0.3 millisecond and less.

PARAMAGNETIC RESONANCE OF CdTe:Mn AND CdS:Mn. John Lambe, Scientific Laboratory, Ford Motor Company, Dearborn, Michigan, and Chihiro Kikuchi, Department of Nuclear Engineering, University of Michigan, Ann Arbor, Michigan (Received February 15, 1960).

Paramagnetic resonance absorption experiments have been carried out on CdTe and CdS crystals containing manganese. In CdTe, the lines are found to be very broad at 300°K. At 4.2°K lines narrow sufficiently to measure parameters, with the result $g = 2.010$, $A = 0.0055$ cm^{-1}, and $3a = 0.0084$ cm^{-1}, where $3a$ is the zero-field splitting. The superhyperfine splitting is the same as for CdS with components spaced at 1.4 gauss, giving $A_{Cd} = 2.6 \times 10^{-4}$ cm^{-1}. In CdS an interaction with conduction electrons is found which can broaden lines at 300°K. At low temperatures an anomalous spectrum is found in CdS with $D = 0.0295$ cm^{-1}.

IMPURITY CONDUCTION IN TRANSMUTATION-DOPED p GERMANIUM. H. Fritzsche, Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois, and M. Cuevas, Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received April 4, 1960).

The Hall coefficient and the resistivity of germanium single crystals bombarded with slow neutrons were measured between 1.2 and 300°K. Slow-neutron capture and subsequent nuclear transmutation produce majority impurities, gallium atoms, and compensating impurities, arsenic and selenium atoms. p-type samples with a gallium concentration ranging from 8×10^{14} to 2×10^{17} per cc with a fixed compensation ratio of 0.40 were thus prepared and the impurity conduction was studied as a function of the average distance between the majority impurities. The effective radius a of the acceptor ground-state wave function is 90.1 A according to Miller's theory of impurity conduction, whereas $a = 40$ A according to Twose's theory. The latter value agrees well with the effective

radius of the Kohn-Schechter acceptor wave function. The activation energy of impurity conduction changes slowly with impurity concentration from 3.5×10^{-4} to 5.9×10^{-4} ev and agrees well with the predictions of Miller's theory for gallium concentration below 5×10^{15} per cc. Measurements on samples which contain different dislocation densities but identical impurity concentrations show that up to 10^4 dislocations per cm^2 do not affect impurity conduction.

MAGNETO-ATTENUATION OF SOUND IN SEMIMETALS: LONGITUDINAL WAVES. Michael J. Harrison,* Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received April 15, 1960).

The calculation of the magnetic field dependence of ultrasonic attenuation in a semimetal is discussed on a simple model of its band structure. The results are applied to the case where the electron and hole mean free paths are large compared to the wavelength of sound. A series of oscillations and a large peak in the attenuation as a function of magnetic field are derived. The oscillations are geometric resonances of the type previously derived for metals, and the large peak is associated with the presence of density waves in the electron-hole carrier gas. The theoretical results are discussed, compared with experimental data, and found to agree semiquantitatively with the latter.

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ELASTIC CONSTANTS OF CADMIUM FROM 4.2°K TO 300°K. C. W. Garland and J. Silverman, Department of Chemistry and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received March 31, 1960).

The adiabatic elastic constants of cadmium single crystals have been measured by an ultrasonic pulse technique. The values extrapolated to 0°K are: $c_{11}=13.08$, $c_{33}=5.737$, $c_{44}=2.449$, $c_{12}=4.048$, $c_{13}=4.145$ in units of 10^{11} dynes/cm^2. A Debye characteristic temperature, θ_0, of 213° ± 1°K has been calculated from these 0°K elastic constants. The temperature dependence of the linear compressibilities, $K_{\parallel}$ and $K_{\perp}$, has also been calculated.

PRESSURE VARIATION OF THE ELASTIC CONSTANTS OF SODIUM. William B. Daniels,* Case Institute of Technology, Cleveland, Ohio (Received April 6, 1960).

The pressure variation of the single-crystal elastic constants of sodium has been measured using a modified ultrasonic pulse echo method. The values found for the pressure derivatives of the elastic constants are: $dC_{44}/dP=1.63$, $dC'/dP=0.226$, $dB_S/dP=3.60$. The notation $C'=(C_{11}-C_{12})/2$ and $B_S=(C_{11}+C_{12})/3$ has been used. The experimental observation that the elastic anisotropy ratio C/C' ($C\equiv C_{44}$) does not depend on pressure indicates that one can positively neglect interaction of ion cores as a contribution to the elastic stiffnesses of sodium. The results are interpreted in terms of Fuchs' theoretical calculation of the Coulomb contribution to the shear stiffnesses of the alkali metals. The interpretation indicates that as sodium is compressed, the value of the electronic wave function at the boundaries of the atomic polyhedra increases more rapidly than $\Omega^{-1/2}$, where Ω is the volume of the atomic polyhedron. The volume variation of the value of the wave function of the lowest electronic state at the boundaries of the atomic polyhedra is found to be

$$\left[\frac{d\ln\mu_0(r_s)}{d\ln\Omega}\right]_{\Omega=\Omega_0}=-0.27.$$

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APPLICATION OF WAVE FUNCTIONS CONTAINING INTERELECTRON COORDINATES. II. APPROXIMATE ENERGY LEVELS OF ATOMS. Peter Walsh, Lamp Division, Research Department, Westinghouse Electric Corporation, Bloomfield, New Jersey, and Sidney Borowitz, Physics Department, New York University, New York, New York (Received April 12, 1960).

This paper explores further the use of interelectron coordinates in constructing atomic wave functions. A simple method is developed for constructing wave functions of this type which yields surprisingly good values for the energy of a variety of atomic systems, in zero order. The Hamiltonian for the system is split into an unperturbed part, which is separable and which contains the interelectron potentials as well as

the electron-nucleus potentials, and into a perturbing term which is always finite and which vanishes whenever an electron is far from the nucleus. The zero-order energies corresponding to this splitting of the Hamiltonian are at least an order of magnitude better for the light atoms than the energies given by the usual Thomas-Fermi theory, and are considerably better than the energies calculated with hydrogenic functions alone in first order.

VARIATIONAL TREATMENT OF ELECTRON-HYDROGEN ATOM ELASTIC SCATTERING. Sydney Geltman, National Bureau of Standards, Washington, D. C. (Received March 8, 1960; revised manuscript received April 18, 1960).

The Hulthén-Kohn variational method is applied to the elastic scattering of electrons by hydrogen atoms. The trial function used is of a nonseparable form, allows for the virtual excitation of the 2*s* and 3*s* states, and contains a scaling parameter which is allowed to vary. The resulting scattering lengths and S, P, and D singlet and triplet phase shifts are compared with the results of other calculations, and reasons are given for expecting them to be of improved accuracy.

APPROXIMATE RELATIVISTIC COULOMB SCATTERING WAVE FUNCTION. W. R. Johnson and C. J. Mullin, University of Notre Dame, Notre Dame, Indiana (Received April 4, 1960).

A relativistic scattering wave function which is valid to second order in αZ for all electron velocities is developed. The wave function has as its dominant term the Sommerfeld-Maue function. The computational simplicity of the closed-form Sommerfeld-Maue wave function is retained in this modified Sommerfeld-Maue function.

Applications of the wave function to the calculation of cross sections for the scattering of polarized electrons and bremsstrahlung production at the short-wave limit of the spectrum are given. A large first order correction to the Sauter-Fano formula for the bremsstrahlung cross section is obtained.

PRECISION DETERMINATION OF THE SLOW-NEUTRON ABSORPTION CROSS SECTION OF B^{10}. G. J. Safford, T. I. Taylor, B. M. Rustad, and W. W. Havens, Jr., Columbia University, New York, New York, and Brookhaven National Laboratory, Upton, New York (Received April 1, 1960).

The total neutron cross section of B^{10} has been measured at neutron energies between 0.00291 ev and 0.1 ev with the Columbia University crystal spectrometer. The B^{10} absorption cross section was obtained by subtracting the relatively small, known, scattering cross section from the measured total cross section. Two samples enriched to 92.84 ± 0.06 atom % and 99.88 ± 0.01 atom % B^{10} were used for these measurements. The B^{10} absorption cross section obtained with the 92.84% B^{10} sample followed the relation $E^{1/2}\sigma_a = 612.3 \pm 2.4$ barns $ev^{1/2}$ and yielded a value of 3849 ± 15 barns at 0.0253 ev. For the 99.88% B^{10} sample, the B^{10} absorption cross section obeyed the relation $E^{1/2}\sigma_a = 608.9 \pm 2.3$ barns $ev^{1/2}$ and gave a value of 3828 ± 15 barns at 0.0253 ev. A sample of Argonne-Brookhaven standard normal boron, 19.8 ± 0.1% B^{10}, was measured simultaneously. The resulting absorption cross section for normal boron was $\sigma_a(0.0253 \text{ ev}) = 764 \pm 3$ barns and obeyed the relation $E^{1/2}\sigma_a = 121.5 \pm 0.5$ barns $ev^{1/2}$. The average of the results for the two enriched samples yields $\sigma_a(B^{10}) = 3838 \pm 11$ barns at 0.0253 ev and $E^{1/2}\sigma_a(B^{10}) = 610.5 \pm 1.7$ barns $ev^{1/2}$. The ratio of the above values of the absorption cross sections for B^{10} and normal boron yields a value of 19.9 ± 0.1% B^{10} for the normal sample which is in good agreement with mass spectrometer determinations.

STUDY OF THE GAMMA-RAY AND INNER BREMSSTRAHLUNG TRANSITIONS IN THE DECAY OF TIN-113. Willie E. Phillips* and John I. Hopkins,† Vanderbilt University, Nashville, Tennessee (Received September 28, 1959; revised manuscript received April 26, 1960).

A fast-slow coincidence spectrometer was used to study the decay of Sn^{113}. A 650-kev gamma ray found in the single-channel spectrum was shown to be in coincidence with the In K x ray both in the fast and in the slow coincidence spectra. This confirmed the existence of a 650-kev level in In^{113} fed by a weak orbital capture branching. From the relative intensi-

ties of the 257-kev gamma rays and the 650-kev gamma rays (16.4 ± 1.9:1), it was determined that the spin of the 650-kev level is 5/2 and the parity is odd. The absence of positrons and arguments from log(ft) classifications indicate a spin of 3/2 and even parity for the ground level of Sn^{113} and a decay energy in the range of 0.8 to 1.6 Mev. The shape of the corrected inner bremsstrahlung spectrum indicated a decay energy of 1.3 ± 0.3 Mev.

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STRIPPING EFFECTS IN THE REACTIONS $C^{12}(He^3, p)N^{14}$ AND $C^{12}(He^3, d)N^{13}$ AT 13.9 Mev. J. R. Priest,* D. J. Tendam, and E. Bleuler, Department of Physics, Purdue University, Lafayette, Indiana (Received April 1, 1960).

Angular distributions have been measured for the reactions $C^{12}(He^3, p)N^{14}$ and $C^{12}(He^3, d)N^{13}$ using the cyclotron 14-Mev He^3 beam. Assuming that the (He^3, p) reaction proceeds by the stripping of a deuteron in either its singlet or triplet state from the He^3 nucleus, reasonable fits to the angular distributions were obtained using a $[j_L(Qr_0)]^2$ dependence. For the transitions to the 1^+, $T=0$ ground and second excited states of N^{14}, $L=2$, $r_0=6.0$ f and $L=0$, $r_0=5.7$ f, respectively. This is in accord with a predominantly $^3D_1$ ground state and predominantly $^3S_1$ second excited state configuration for N^{14}. For the transition to the 0^+, $T=1$ first excited state, $L=0$ and $r_0=5.7$ f. Butler's stripping theory for the (He^3, d) reaction with $l_p=1$ and $r_0=5.8$ f accounts reasonably well for the $C^{12}(He^3, d)N^{13}$ angular distribution.

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ANGULAR DISTRIBUTIONS FOR $C^{12}(\alpha, p)N^{15}$ AT 16.1-19.0 Mev AND $F^{19}(\alpha, p)Ne^{22}$ AT 18.9 Mev. J. R. Priest,* D. J. Tendam, and E. Bleuler, Department of Physics, Purdue University, Lafayette, Indiana (Received April 1, 1960).

Angular distributions have been measured for the reactions $C^{12}(\alpha, p)N^{15}$ and $F^{19}(\alpha, p)Ne^{22}$ using the cyclotron 19-Mev alpha-particle beam. The $C^{12}(\alpha, p_0)N^{15}$ data at 18.0, 17.1, and 16.1 Mev differ markedly from the 19.0-, 18.7-, and 18.3-Mev data; the latter three are very similar to each other. The differential excitation function at 31.8° (lab) shows a resonance at 17.5 Mev with a width at half-maximum of ~2 Mev. Steep backward peaking is observed in the 16.1-Mev data. Some of the features of the 17.1- and 18.0-Mev data can be represented by Butler's formula for (α, p) reactions using $l=1$ and $r_0=5.9$ f and 5.5 f, respectively. For the $F^{19}(\alpha, p)Ne^{22}$ data the cross sections for the transition to the ground state of Ne^{22} are factors of 10 to 20 less than the cross sections for the transition to the first excited state of Ne^{22}. This inhibition, together with the $F^{19}(n, d)O^{18}$ data, may be explained by assuming that the two last neutrons of F^{19} are not disturbed in the direct interactions and that they are in a $(d_{5/2}{}^2)_0$ configuration in Ne^{22}, in a $(d_{5/2}{}^2)_2$ or $(s_{1/2}{}^2)_0$ configuration in F^{19}. Using $r_0=5.1$ f, Butler's formula accounts reasonably well for the forward peak and the location of the minima of all three angular distributions for the region of angles less than 90°. The angular distributions are all peaked in the backward angles.

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ELASTIC SCATTERING OF 13.5- AND 15-Mev DEUTERONS BY NUCLEI. Nikola Cindro* and N. S. Wall, Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received April 4, 1960).

15-Mev deuterons from the MIT cyclotron were elastically scattered by a number of elements, including Al, Ti, Fe, Cu, Ni, Rh, Pd, Sn (natural and monoisotropic), Ta, Au, and Pb. The angular distributions were measured in the MIT cyclotron 24-in. scattering chamber, by means of a NaI(Tl) scintillation detector. The crystal was thick enough to stop 15-Mev deuterons but too thin to stop protons of the same energy, thus permitting measurement of the (d, d) cross sections for light elements, where (d, p) reactions play an important role. The obtained distributions differ sharply from the Rutherford formula $\sigma_C \propto \csc^4(\theta/2)$; most of them, especially those for medium and lighter elements, show pronounced diffraction-like maxima and minima.

The energy of the beam was degraded to 13.5 Mev and the angular distributions for Ni, Sn

(natural), and Au measured, showing the same general characteristics.

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NEUTRON TRANSFER AND THE TUNNELING MECHANISM IN THE BOMBARDMENT OF GOLD BY NITROGEN. J. A. McIntyre, T. L. Watts, and F. C. Jobes, Yale University, New Haven, Connecticut (Received March 17, 1960).

Measurements have been made of the angular distributions of N^{13} nuclei produced in the neutron-transfer reaction $Au^{197}(N^{14}, N^{13})Au^{198}$. Fifteen different bombarding energies were studied. It is found that the results can be described in terms of the transfer of a neutron from an N^{14} nucleus moving along a classical trajectory. In particular, it is found that N^{14} trajectories with distances of closest approach of $(12.7 \pm 0.5) \times 10^{-13}$ cm give the largest contribution to the transfer process. Further, both the angular and energy dependence of the N^{13} production have been compared with the dependences predicted by the tunneling theory of Breit and Ebel; both experimental dependences have been found to be in agreement with this theory.

INTERNAL CONVERSION ELECTRONS FOLLOWING COULOMB EXCITATION OF HIGHLY DEFORMED ODD-*A* NUCLEI. E. M. Bernstein* and R. Graetzer, Physics Department, University of Wisconsin, Madison, Wisconsin (Received March 23, 1960).

The internal conversion electrons emitted following Coulomb excitation of Eu^{153}, Dy^{161}, Dy^{163}, Ho^{165}, Tm^{169}, Lu^{175}, and Ta^{181} have been measured. The relative intensities of the decay transitions from the first two rotational states of these isotopes have been compared with the predictions of the rotational model of Bohr and Mottelson. In general, there is good agreement between the experiment and theory; however, the results for the Dy isotopes indicate some disagreement which is outside the experimental uncertainty. For Eu^{153}, Dy^{163}, and Lu^{175} transitions involving intrinsic states were observed in addition to the rotational transitions. The reduced $E2$ transition probabilities for these intrinsic transitions are appreciably larger than single-particle estimates. The first rotational state in Lu^{176} was observed with a natural Lu target. The data indicate that this transition is predominantly $M1$.

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NUCLEAR SPECTROSCOPY OF NEUTRON-DEFICIENT Lu, Ta, AND Re ISOTOPES. B. Harmatz and T. H. Handley, Oak Ridge National Laboratory, Oak Ridge, Tennessee, and J. W. Mihelich, University of Notre Dame, Notre Dame, Indiana (Received February 17, 1960).

The systematic behavior of nuclear energy levels has been studied with Lu ($Z = 71$), Ta ($Z = 73$), and Re ($Z = 75$) activities produced in the ORNL proton cyclotron. Conversion-electron data are presented for electron-capture decay of $Lu^{170,174m}$, $Ta^{173-178}$, and $Re^{179,181}$. Level schemes are proposed based on these and on previously published transition data for $Lu^{169,171}$. The properties of odd-A nuclei in the strongly deformed region of odd-N numbers 95-107 are discussed in connection with predictions of Mottelson and Nilsson. Two activities, Lu^{174m} and Re^{179} (~20 min), are previously unreported.

SHAPE OF BETA SPECTRA OF ONCE FORBIDDEN TRANSITIONS IN THE DECAYS OF Ga^{72}, La^{140}, Eu^{152}, Eu^{154}, AND Sb^{124}. L. M. Langer and D. R. Smith, Physics Department, Indiana University, Bloomington, Indiana (Received April 4, 1960).

Magnetic spectrometer studies were made on the beta spectra of Ga^{72}, La^{140}, Eu^{152}, Eu^{154}, and Sb^{124}. The highest energy beta group of each of these transitions has an abnormally long comparative half-life and probably is a once forbidden decay from a 3- initial state to a 2+ final state. For all, the highest energy beta spectrum was found to have a nonstatistical nonunique shape. On the assumption that the B_{ij} matrix element dominates the decay, the experimental shape factors were fitted empirically by $C = q^2 + \lambda p^2 + D$. The best fits were obtained for values of D equal to 15, 10, 5, 20, and 15 for Ga^{72}, La^{140}, Eu^{152}, Eu^{154}, and Sb^{124}, respectively.

INTERACTION OF HIGH-ENERGY PROTONS AND ALPHA PARTICLES WITH IODINE-127. Inge-Maria Ladenbauer* and Lester Winsberg, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 2, 1960; revised manuscript received April 29, 1960).

Iodine was bombarded with protons ranging in energy from 0.25 to 6.2 Bev and with 0.25-, 0.50-, and 0.72-Bev alpha particles. Reactions of the type (p,pxn), $(p,2pxn)$, $(p,p\pi^+)$, $(p,p2\pi^+)$, $(p,n\pi^-)$, and $(\alpha,\alpha xn)$ to produce iodine, tellurium, antimony, and cesium isotopes were investigated. Upper limits in the range 0.01 to 0.1 mb were found for cross sections of reactions to produce Sb^{127} and Cs^{127}. For Te^{127}, upper limits in the range of 1 to 2 mb were found. This and other studies of reactions in which the product has the same mass number as that of the target are discussed in terms of the initial interaction. At all incident energies studied, the cross section for the formation of I^{126} via the (p,pn) or $(\alpha,\alpha n)$ reaction is significantly higher than that of the other (p,pxn) or $(\alpha,\alpha xn)$ reactions, with the possible exception of $(\alpha,\alpha 4n)$. The (p,pn) and probably the $(\alpha,\alpha n)$ reactions appear to be due primarily to knock-on collisions with surface neutrons. The excitation functions for the production of iodine isotopes by proton bombardment decrease between 0.25 and 0.72 Bev but remain relatively constant for higher energies. The $(p,2pxn)$ reactions show a similar effect, but with the excitation functions becoming constant at about 2 Bev. These results are compared with Monte Carlo calculations of the proton-initiated nucleon cascade and of the subsequent evaporation of light particles.

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INTERACTION OF HIGH-ENERGY PROTONS WITH INDIUM. David R. Nethaway and Lester Winsberg, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 2, 1960; revised manuscript received April 29, 1960).

Indium was bombarded with protons ranging in energy from 1.0 to 6.2 Bev. Reactions of the type (p,pxn), $(p,2pxn)$, (p,p'), and $(p,p\pi^+)$ that produce isotopes of indium and cadmium were investigated. The excitation functions are constant within experimental error in this energy region with possible exceptions for Cd^{115} and In^{115m} at 1.0 Bev. These results are compared with two types of calculation. In one treatment, the nucleus is considered to be a degenerate Fermi gas of nucleons. The cross sections that were calculated with this nuclear model at 2 Bev are much smaller than the experimental values. There is good agreement at 1 Bev for nuclides with a mass number less than 113. The second treatment takes into account the shell structure of In^{115}. The latter calculation for the (p,pn), reaction was in good agreement with the experimental results at 4.1 and 6.2 Bev. The comparison of the experimental results with the calculated values is discussed in terms of the adequacy of the calculations.

MOMENTUM AND ASYMMETRY SPECTRUM OF μ-MESON DECAY. Richard J. Plano, Columbia University, New York, New York (Received April 8, 1960).

The beta decay of the positive μ meson was studied using a liquid hydrogen bubble chamber in a magnetic field of 8800 gauss. An analysis of 9213 events used in the momentum spectrum yielded $\rho = 0.780 \pm 0.025$. This number includes the internal radiative corrections and is to be compared directly with the 0.75 predicted by two-component theory. The analysis of 8354 events used in the asymmetry spectrum gave for the magnitude of the asymmetry $|\xi| = 0.94 \pm 0.07$ and for the shape parameter $\delta = 0.78 \pm 0.05$.

PHOTOPRODUCTION OF π^0 MESONS FROM HYDROGEN IN THE REGION 900 TO 1200 Mev. H. E. Jackson,* J. W. DeWire, and R. M. Littauer, Cornell University, Ithaca, New York (Received April 4, 1960).

The reaction $\gamma + p \rightarrow \pi^0 + p$ has been studied in three adjacent 100-Mev energy intervals between 900 and 1200 Mev and at pion center-of-mass angles of 47°, 90°, and 125°. The reaction was observed as a coincidence between the recoil proton and one of the photons from the meson's decay. The kinematics were determined by the energy of the incident photon and the angle of the recoil proton. The differential cross sections at the forward and backward angles show pronounced maxima near 1050 Mev,

while the 90° cross sections decrease slowly with energy. The estimated total cross sections suggest a narrow maximum near 1050 Mev. These features are consistent with the previously proposed existence of a resonant state in the pion-nucleon system of total angular momentum 5/2.

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PROTON-ANTIPROTON ANNIHILATION IN PROTONIUM. Bipin R. Desai, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received April 1, 1960).

Using the model for the nucleon-antinucleon interaction proposed by Ball and Chew, we have calculated the capture rates for the various eigenstates of protonium—the bound system of a proton and an antiproton. It is found that these rates depend sensitively on spin, isotopic spin, and total angular momentum eigenvalues of protonium, not just on orbital angular momentum, as is usually assumed. The average capture rates for the nS and nP states are $5.3\times10^{18}/n^3$ and $4.3\times10^{14}/n^3$ sec^{-1}, respectively. This P capture rate is two orders of magnitude larger than in the case of the $(K^- - p)$ atom because of the relatively long range of interaction in the Ball-Chew model. The problem of the Stark effect collisions, studied by Day, Snow, and Sucher in connection with the $(K^- - p)$ atom, is therefore re-investigated and at the same time we have considered certain important effects which were not considered by these authors. Rough calculations indicate that for protonium also the capture will take place predominantly from S states.

PION MULTIPLICITY IN NUCLEON-ANTINUCLEON ANNIHILATION. Bipin R. Desai, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received April 1, 1960).

In the annihilation problem we have considered the influence of the Ball-Chew model, according to which, at low energies, only a few of the eigenstates of the nucleon-antinucleon system need be considered. The effect of the selection rules that forbid certain pion multiplicities is thereby examined. The energies considered are 50 Mev, 140 Mev, and 0 Mev in the case of protonium—the bound system of a proton and an antiproton. To obtain the multiplicity, we have used the Fermi statistical model but have introduced Lorentz-invariant phase space, thus defining a new interaction volume. It is found that due to selection rules there is a substantial change in the number distribution of the outgoing pions. At 140 Mev and in the case of protonium the two-pion production is decreased considerably. The zero-prong events for the $p\bar{p}$ annihilation are suppressed by about a factor of two for annihilations at rest in the case of protonium compared to the corresponding events for annihilations in flight. The over-all average multiplicity is unchanged, however. The value of the newly defined interaction volume, in units of Fermi volume, for $p\bar{p}$ and $N\bar{p}$ annihilations (where N denotes an "average" nucleon) should be ~10 in order to fit the observed multiplicities.

THRESHOLD EFFECTS IN THREE-BODY CHANNELS. Luciano Fonda, Institute for Advanced Study, Princeton, New Jersey and Roger G. Newton, Department of Physics, Indiana University, Bloomington, Indiana (Received April 5, 1960).

The possibility of obtaining threshold anomalies in reactions leading to three-particle channels is studied in detail. It is found that a threshold cusp or rounded step exists in reactions whose final three-body channels have at least one particle in common. The effect appears as a function of the momentum of the common particle while the total energy is fixed.

ANELASTICITY OF p-p COLLISIONS AT 2.7 Bev. W. M. Bugg and D. T. King, The University of Tennessee, Knoxville, Tennessee (Received April 11, 1960).

By following the tracks of particles in a direction generally contrary to that of the incident 2.7-Bev proton beam, a series of observations interpreted as p-p collisions has been made in nuclear emulsions. The angular and momentum distributions of particles outgoing from these interactions, and hence the relative occurrence of p-p collisions with different pion multiplicities, are in reasonable accord with foregoing work. An attempt is made to distinguish triple

pion production processes from those of double production. Even though the center-of-mass kinetic energy distributions for the respective production modes are markedly different, the corresponding anelasticity characteristics show a general similarity.

BARYON MASS SPECTRUM. Katsumi Tanaka, Argonne National Laboratory, Lemont, Illinois (Received March 14, 1960).

It is assumed that the π-baryon interactions are universal and the K-baryon interactions account for the large mass differences between baryons. If further all baryon spins are $\frac{1}{2}$ and the K spin is zero, the (Σ,Λ) parity is even, the (Σ,Λ) mass difference can be neglected, and the present baryon spectrum and its isotopic spin assignments are correct, then to all orders in the π-baryon coupling constant and to the second order in the K-baryon coupling constants, one can obtain the essential feature of the observed mass spectrum. The magnitudes of the K-coupling constants that yield this mass spectrum are crudely estimated.

PION-PION INTERACTIONS IN τ AND τ' DECAYS. R. F. Sawyer and K. C. Wali, University of Wisconsin, Madison, Wisconsin (Received April 11, 1960).

The final-state interactions in τ and τ' decays of K^+ mesons are studied by means of a Mandelstam representation. It is assumed that only the S-wave pion-pion scattering amplitude is large enough to have an appreciable imaginary part. Coupled, linear, singular, integral equations are found for the amplitudes describing $K^+ + \pi - \pi + \pi$ in the physical and unphysical regions. From the solutions to these equations the τ and τ' decay matrix elements may be constructed. In an approximation the equations are solved in terms of pion-pion phase shifts. Comparison with experiment is made using Chew and Mandelstam's solution of the corresponding nonlinear equations for π-π scattering. Consistency with experiment is found for values of the coupling constant implying repulsive $T=0$ and $T=2$ phase shifts. Independently of the approximation the equations show that the implications in τ and τ' decay of the rule, $\Delta T = \frac{1}{2}, \frac{3}{2}$, are identical with those of the $\Delta T = \frac{1}{2}$ rule. Hence the energy spectra in τ' decay cannot be a critical test of the $\Delta T = \frac{1}{2}$ rule. Also independently of the approximation it is shown that the decay matrix element cannot be expanded in a series of integral powers of the pion kinetic energies.

GROUND AND EXCITED STATES OF A MANY-BODY SYSTEM WITH SINGULAR INTERACTION. Katuro Sawada,* Institute for Advanced Study, Princeton, New Jersey (Received December 14, 1959).

The procedure to get the ground-state energy and excitation spectrum by looking at the normal modes for the simple excitations of a many-body system is extended and applied to a system with singular interaction. It is shown that the singular two-body interaction can be consistently replaced by the so-called reaction matrix in the equations of motion and in the expressions for the energies of ground and excited states.

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μ-MESON DECAY INTO THREE ELECTRONS. M. Bander and G. Feinberg, Department of Physics, Columbia University, New York, New York (Received April 7, 1960).

The decay $\mu \to e^- + e^- + e^+$ via internal conversion is computed using a phenomenological matrix element for the $\mu e \gamma$ interaction. The result is compared with present experimental limits for this process and the results concerning the form factors in the matrix element are discussed. The energy distribution of the emitted electrons is also computed.

IMPLICATIONS OF THE INTERMEDIATE BOSON BASIS OF THE WEAK INTERACTIONS: EXISTENCE OF A QUARTET OF INTERMEDIATE BOSONS AND THEIR DUAL ISOTOPIC SPIN TRANSFORMATION PROPERTIES. T. D. Lee, Columbia University, New York, New York and C. N. Yang, Institute for Advanced Study, Princeton, New Jersey (Received April 11, 1960).

Assuming that all weak interactions are transmitted through an intermediate boson field W, it is shown that the observed $|\Delta \vec{I}| = \frac{1}{2}$ rule and

the small observed mass difference between K_1 and K_2 lead to the conclusion that there exist four W particles: $W^{\pm}$, W^0, and $\overline{W}^0$. Furthermore, a natural assignment of the isotopic spin transformation property of these W particles follows a dual scheme in which the W's behave sometimes as $I=\frac{1}{2}$ and sometimes as $I=1$ particles. Various experimental implications are discussed, including neutrino capture experiments, strong collisions exhibiting apparent nonconservation of strangeness, and strong collisions with apparent lepton production.

PARTIAL-WAVE DISPERSION RELATIONS FOR PION-NUCLEON SCATTERING. William R. Frazer, Institute for Advanced Study, Princeton, New Jersey and Jose R. Fulco, Department of Physics, University of Buenos Aires, Buenos Aires, Argentina (Received April 8, 1960).

Partial-wave dispersion relations for pion-nucleon scattering are derived from the Mandelstam representation. The symmetry of the representation is used to obtain expressions for the discontinuities across the unphysical branch cuts. These expressions involve scattering amplitudes for pion-nucleon scattering and for the process $\pi+\pi \rightarrow N+\overline{N}$. In the approximation of neglecting all but the nearest singularities it is shown that the Chew-Low effective-range formula is a solution to the equations.

SOME INTEGRAL REPRESENTATIONS FOR THE VERTEX FUNCTION. A. Minguzzi and R. F. Streater, CERN, Geneva, Switzerland (Received March 28, 1960).

It is noted that the integral representations for matrix elements derived in three papers, (1) Dyson, (2) Deser, Gilbert, and Sudarshan, and (3) Fainberg, do not give the most general functions satisfying the conditions stated. It is shown that in (1) and (2) the convergence of certain integrals was not treated rigorously, while in (3) there was a misunderstanding of the Jost-Lehmann-Dyson representation.

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